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Lecture Series:

Structural Dynamics

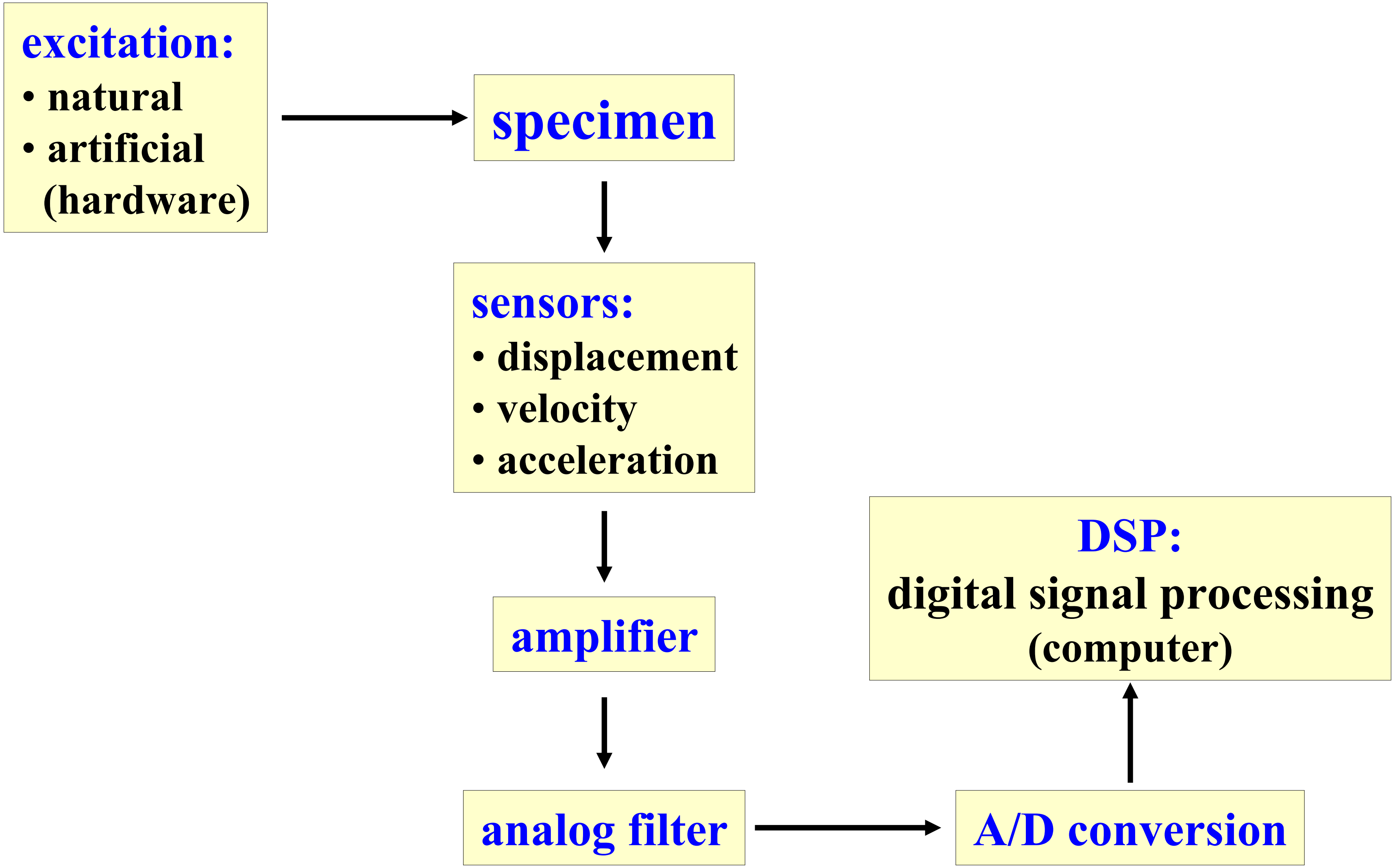
Lecture 14:

Experimental Vibration Analysis

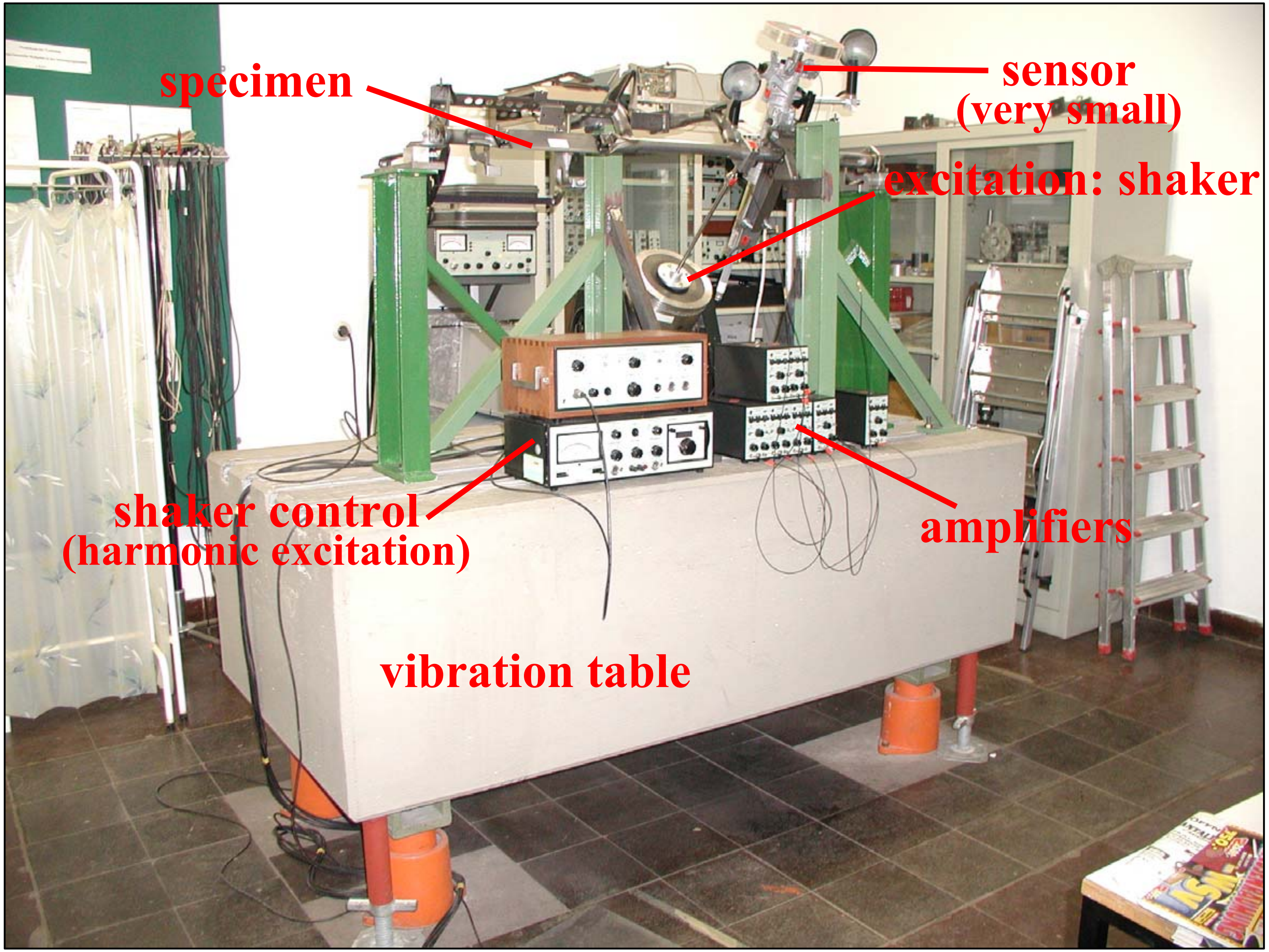
Overview

- **Elements of an experimental setup**
- **Measurement of oscillations:**
  - **generation of excitation**
  - **sensors**
  - **signal amplification**
- **Data analysis:**
  - **data filtering: anti-aliasing, windowing**
  - **system identification**

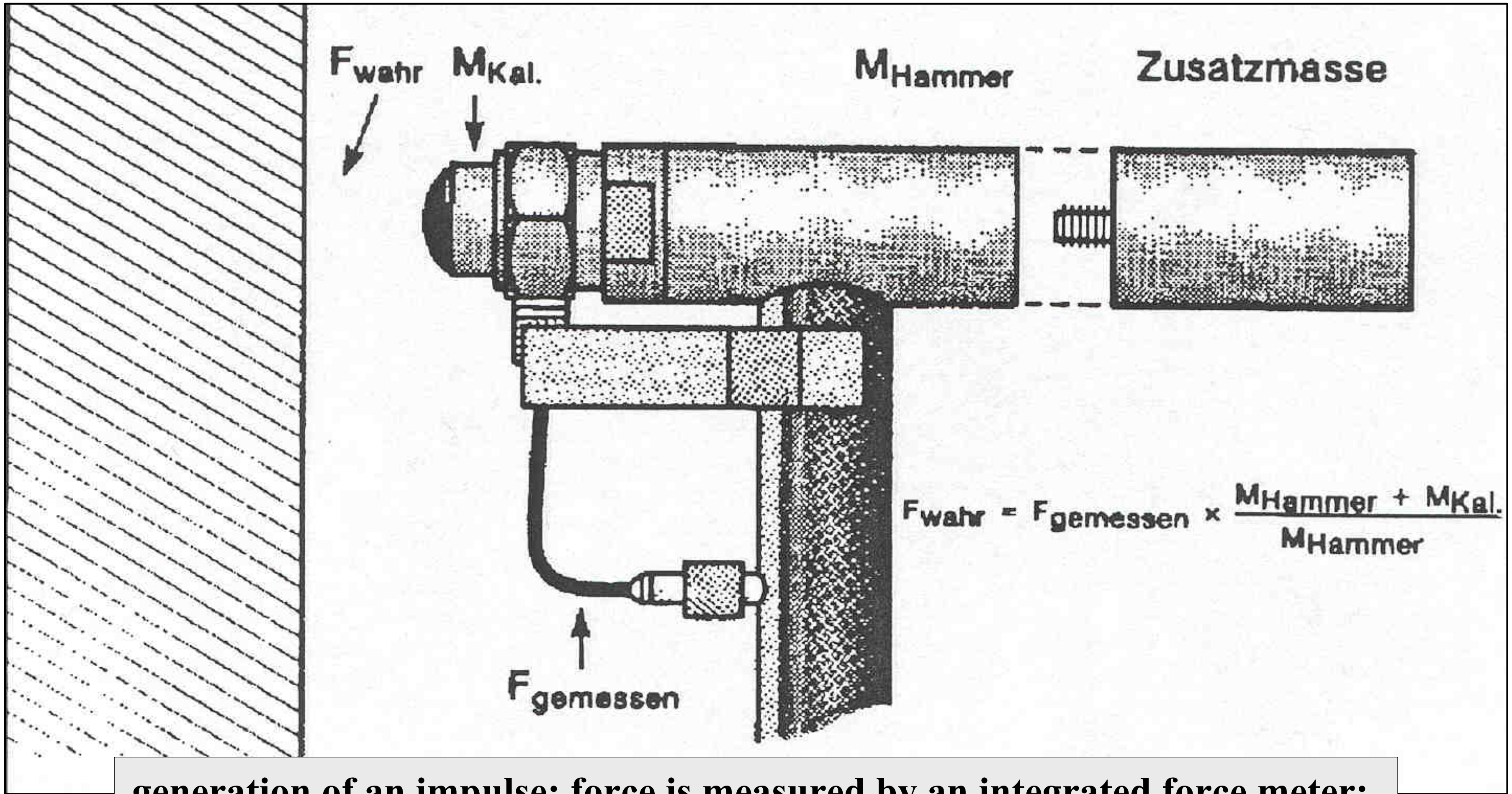
Elements of an Experimental Setup



Example of a Setup in the Lab



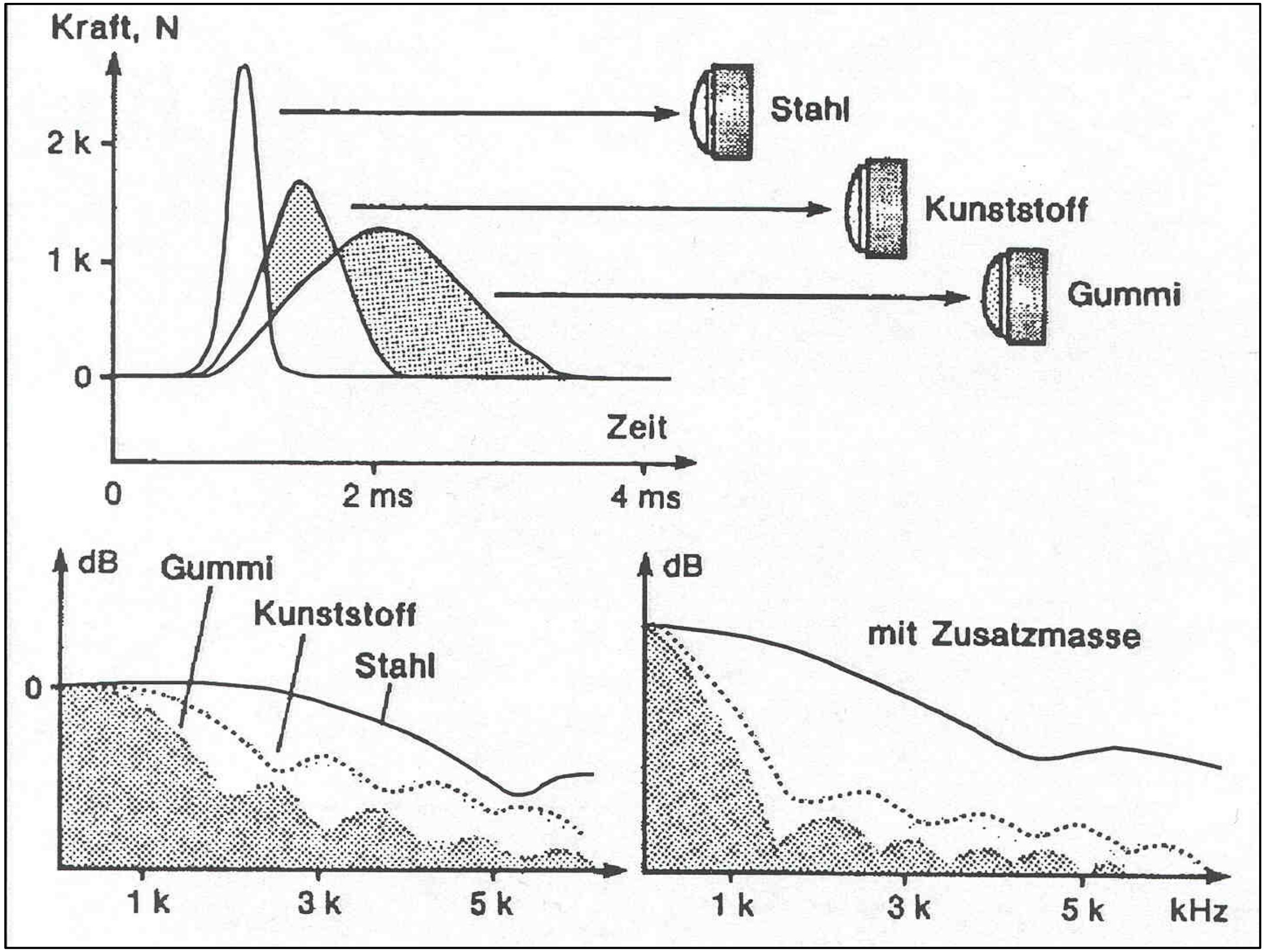
Generation of a Transient Excitation: Impulse Hammer



generation of an impulse: force is measured by an integrated force meter:

- ⇒ calculation of transfer functions.

Impulse Hammer



spectrum can be varied by the use of different contact materials and additional masses:

- **hard material: short impulse**
- **soft material: longer impulse.**

Generation of a Specified Excitation: Shaker

generation of arbitrary excitations
by electro-dynamic means:

• harmonic excitation
• random excitation
• ...

parallel measuring of force:

• generation of transfer functions

Sensors: Overview

power source
- passive: need auxiliary power source
- active: generate directly a voltage

physical quantity
- displacement meter
- acceleration meter
- velocity meter

measurement mode
- needs a fixed reference point
- without a fixed reference point

Inverse Problem of Testing

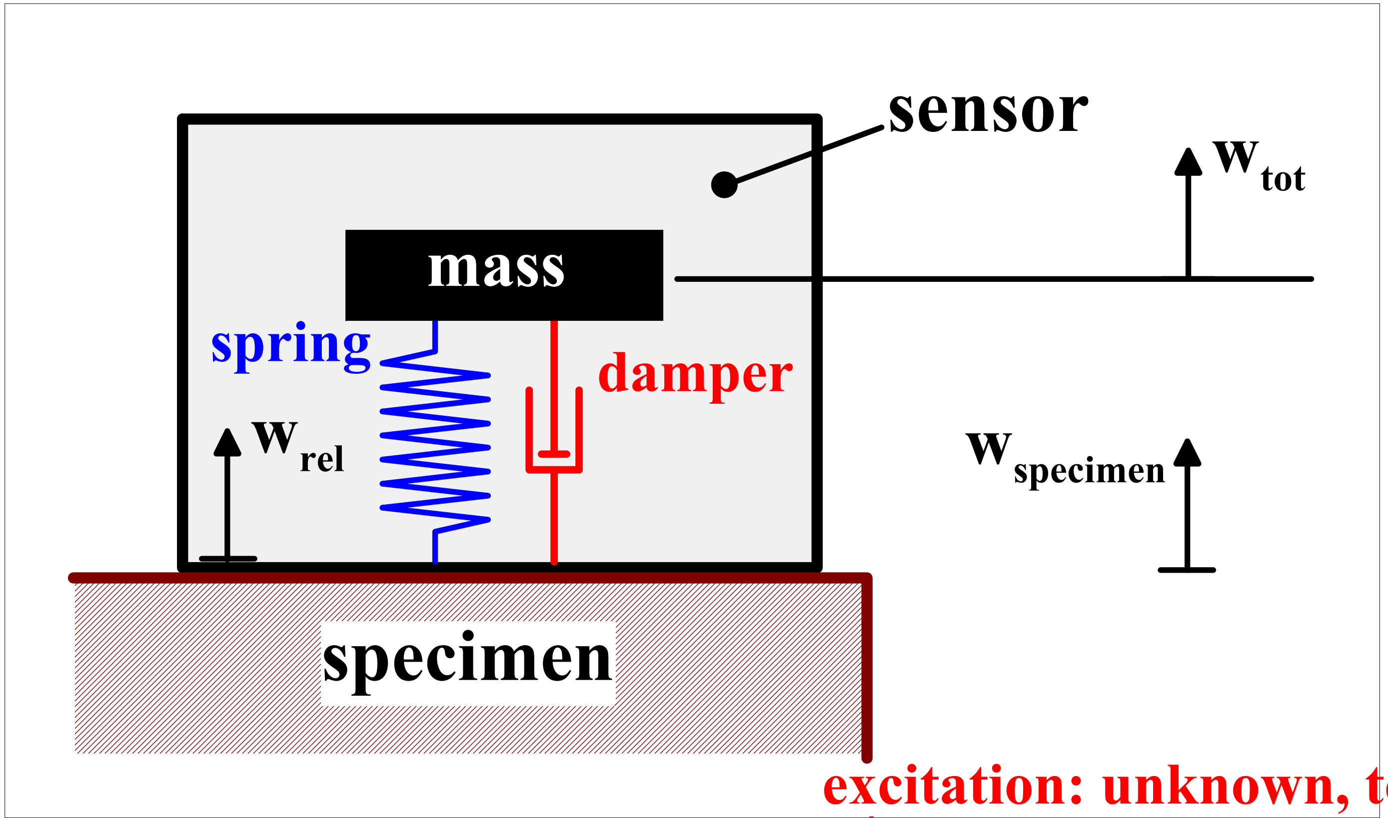
direct problem of calculating the response:

input: excitation	oscillator: structure	output: response
p given	$\hat{u} = \lvert H_{up} \rvert \hat{p}$	u unknown

inverse problem of calculating the excitation:

input: excitation (response of structure)	oscillator: sensor	output: response (response of sensor)
p calculated	$\hat{p} = \lvert H_{up} \rvert^{-1} \hat{u}$	u measured

Seismic Sensor – General Description



excitation: unknown, to be calculated

$$w_{tot} = w_{rel} + w_{specimen}$$

response: measured

Seismic Sensor – Equation of Motion

equation of motion for an SDOF-system under ground acceleration:

$$m\,\ddot{w}_{tot} + c\,\dot{w}_{rel} + k\,w_{rel} = 0$$

equation of motion for the relative displacement:

$$m\,\ddot{w}_{rel} + c\,\dot{w}_{rel} + k\,w_{rel} = -m\,\ddot{w}_{s} = p_{ground}$$

w_s: oscillation of the specimen

Question:

What is the response of the sensor for the "ground motion" of the specimen?

Harmonic Oscillation of Specimen

displacement of the specimen:

$$w_s(t) = \hat{w}_s \sin \Omega t$$

velocity and acceleration:

$$\dot{w}_s(t) = -\hat{w}_s \Omega \cos \Omega t = -\hat{\dot{w}}_s \cos \Omega t$$

$$\ddot{w}_s(t) = -\hat{w}_s \Omega^2 \sin \Omega t = -\hat{\ddot{w}}_s \sin \Omega t$$

$$p_{ground}(t) = m \hat{w}_s \Omega^2 \sin \Omega t$$

Seismic Sensor as Displacement Meter:

displacement of the harmonically excited oscillator:

$$w_{rel}(t) = \frac{m\,\hat{w}_s\Omega^2}{k} V_1 \sin(\Omega t - \Psi) = \left|\underline{H}_{w_{rel}w_s}\right| \hat{w}_s \sin(\Omega t - \Psi)$$

dynamic amplification factor V_1:

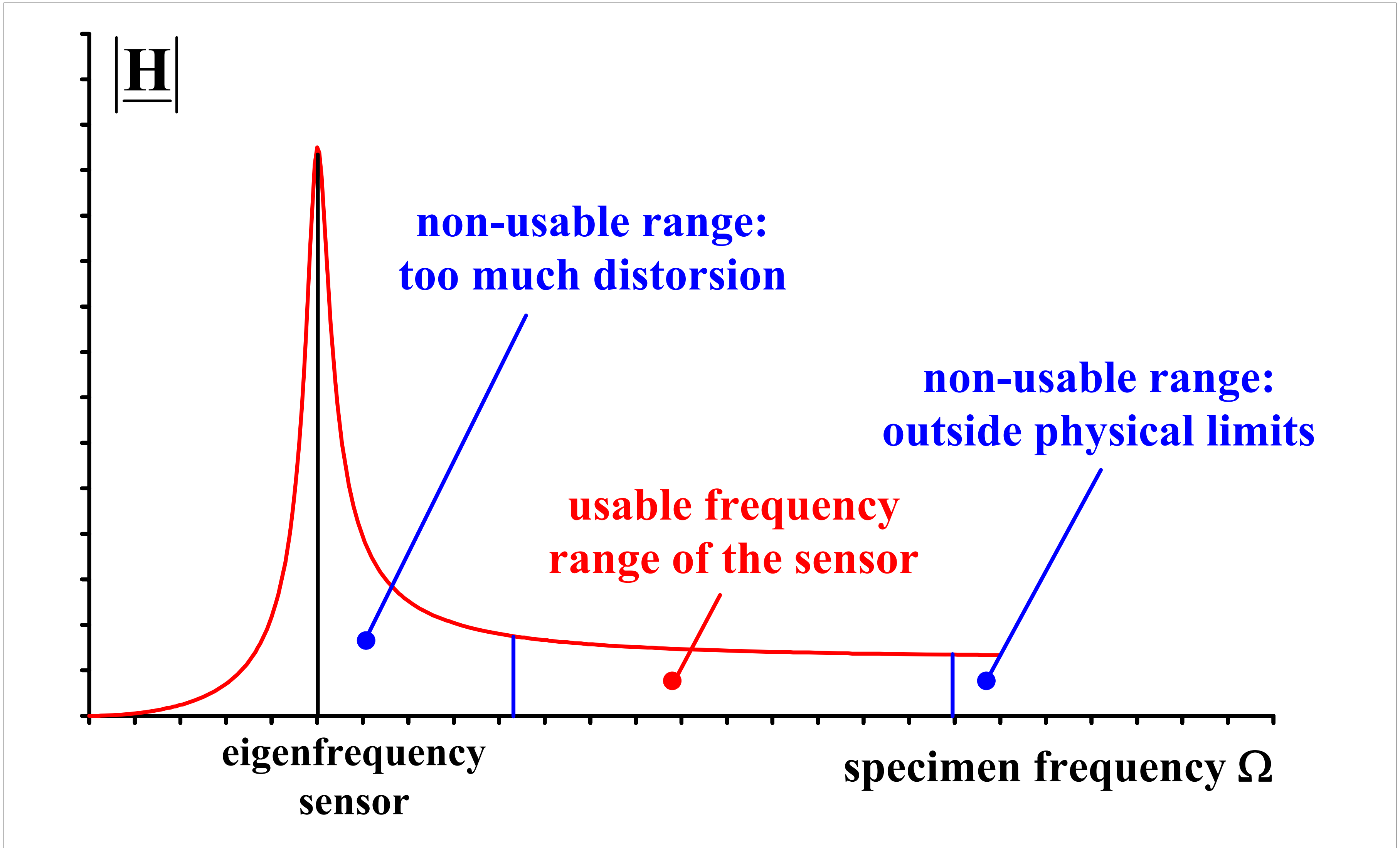
$$V_1(\xi,\eta) = \frac{1}{\sqrt{(2\,\xi\eta)^2 + (1-\eta^2)^2}}$$

$$\Rightarrow \quad \left|\underline{H}_{w_{rel}w_s}\right| = \frac{\eta^2}{\sqrt{(2\,\xi\eta)^2 + (1-\eta^2)^2}}$$

$$\Rightarrow \quad \hat{w}_s = \frac{\hat{w}_{rel}}{\left|\underline{H}_{w_{rel}w_s}\right|}$$

This allows a computation of the displacement of the specimen from the relative displacement within the sensor which is measured.

Frequency Range of the Displacement Meter



Seismic Sensor as Accelerometer

solution for the displacement meter:

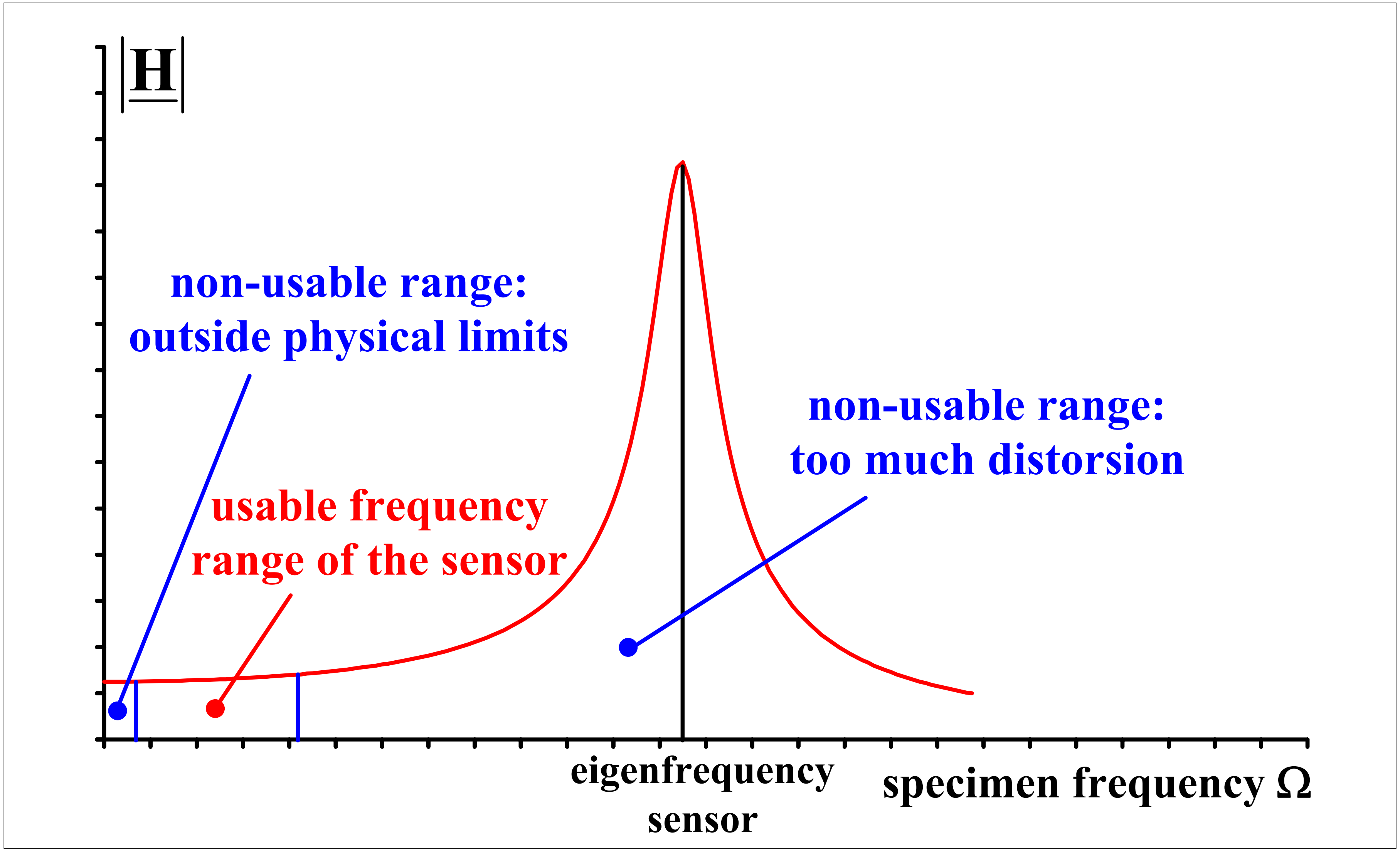
$$w_{rel}(t) = \frac{m\,\hat{w}_s \Omega^2}{k} V_1 \sin(\Omega t - \Psi)$$

$$\hat{\ddot{w}}_s(t) = \hat{w}_s \Omega^2$$

$$w_{rel}(t) = \frac{1}{\omega^2} \hat{\ddot{w}}_s V_1 \sin(\Omega t - \Psi)$$

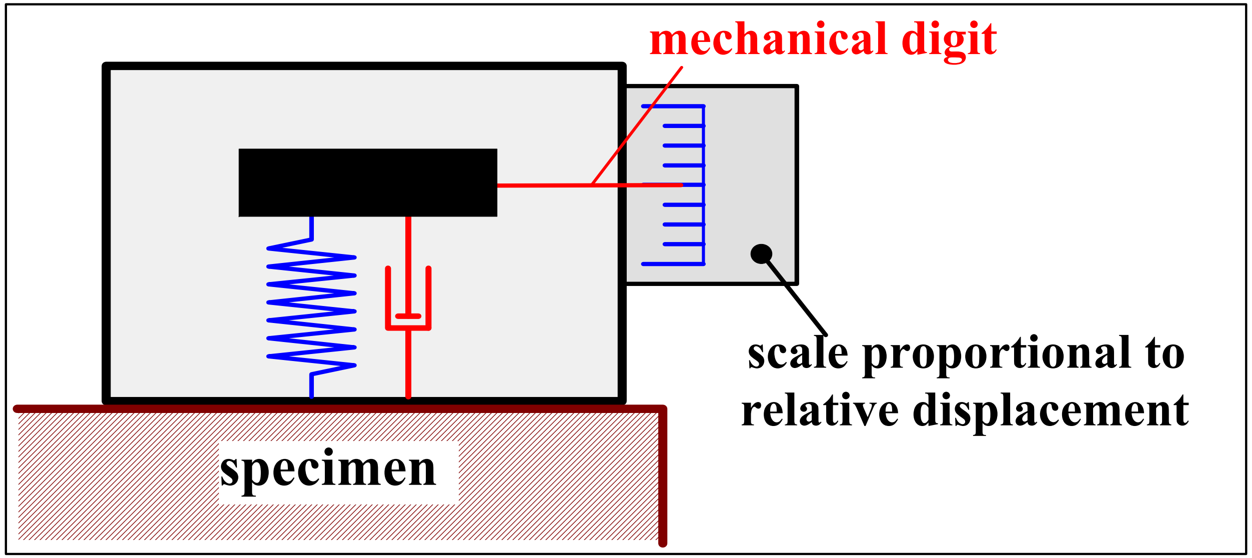
$$\left| \underline{H}_{w_{rel}\ddot{w}_s} \right| = \frac{1}{\omega^2} V_1$$

Frequency Range of the Accelerometer



Data Acquisition from a Sensor

old-time method: mechanical data acquisition

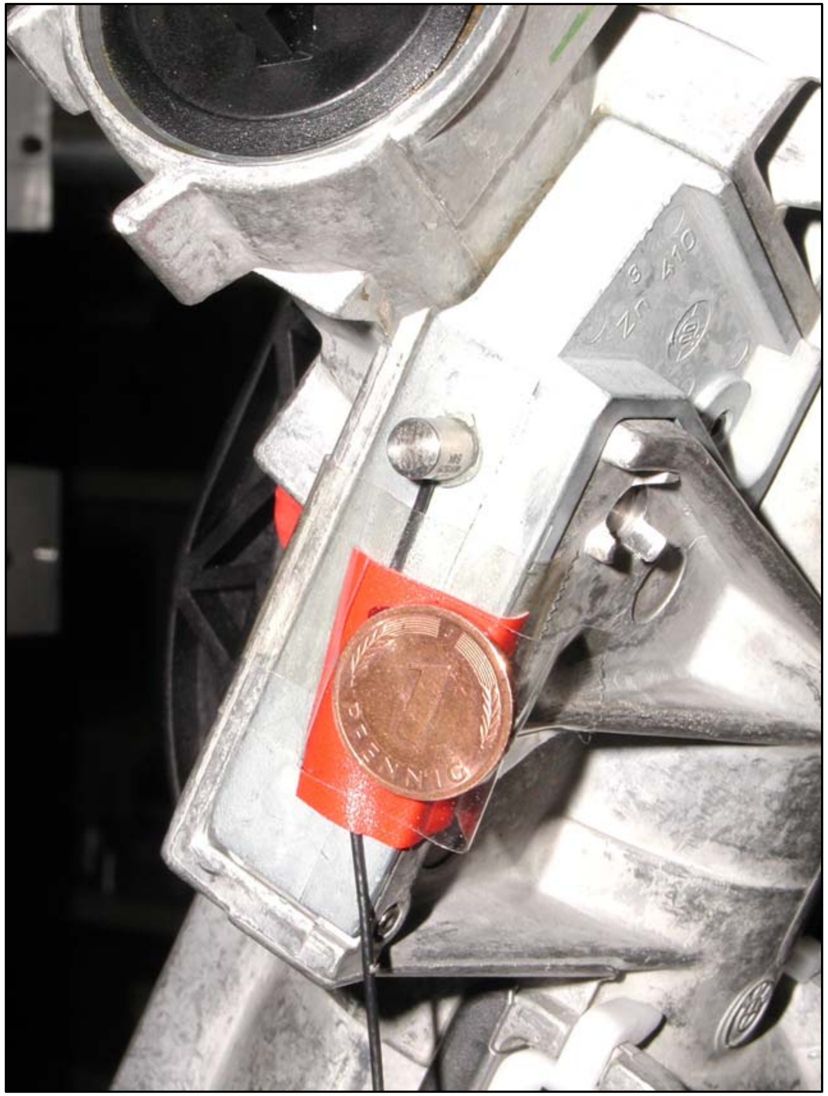


- time domain response is recorded
- no direct insight into the frequency composition of response

➡ signal must be digitized ➡ FFT analysis of discrete sample

➡ relative displacement is transformed into voltage

Piezoelectric Accelerometer



A piezoelectric element (crystal) produces a small electric charge which is proportional to its deformation.

↓

amplification (charge)

↓

conversion into voltage

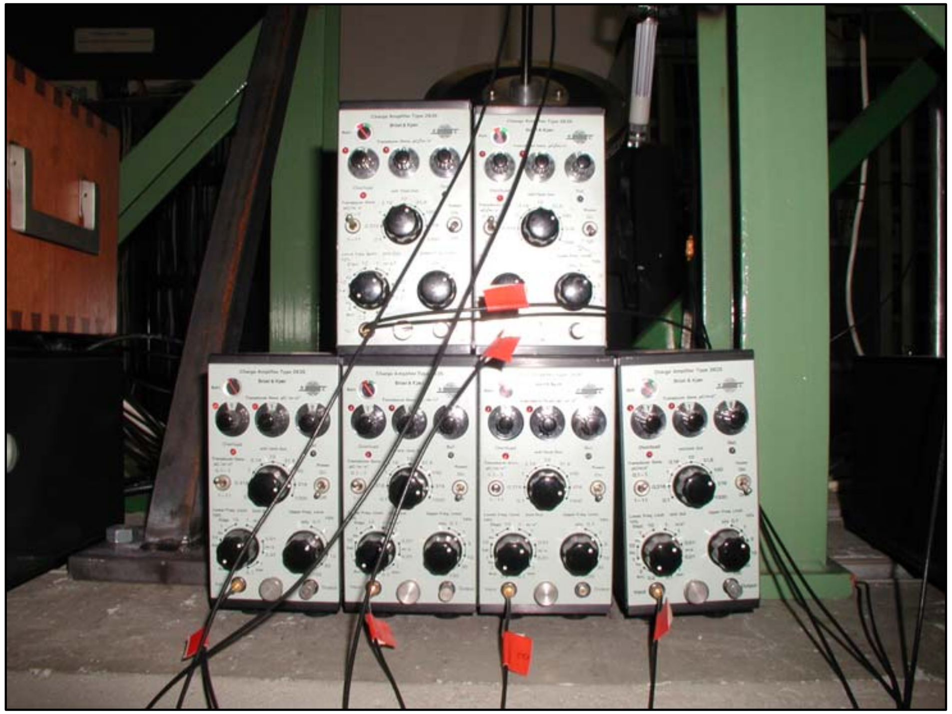
↓

amplification (voltage)

↓

A/D converter

Amplification and A/D Conversion



The analog/digital (A/D) conversion assigns an integer number to each (in principle) continuous voltage value by dividing the entire voltage range of the converter into intervals.

Word-length of the converter:

- 8-bit converter: 256 intervals
- 16-bit converter: 65536 intervals

We get the best resolution if the signal is amplified such that it spans the entire voltage range of the A/D converter.

Some Problems in DSP

electric signal:
voltage
analog signal

→

A/D conversion:
"jitter" problem

→

digital signal
(time domain)

↓

FFT:
"aliasing" effect
"leakage" effect

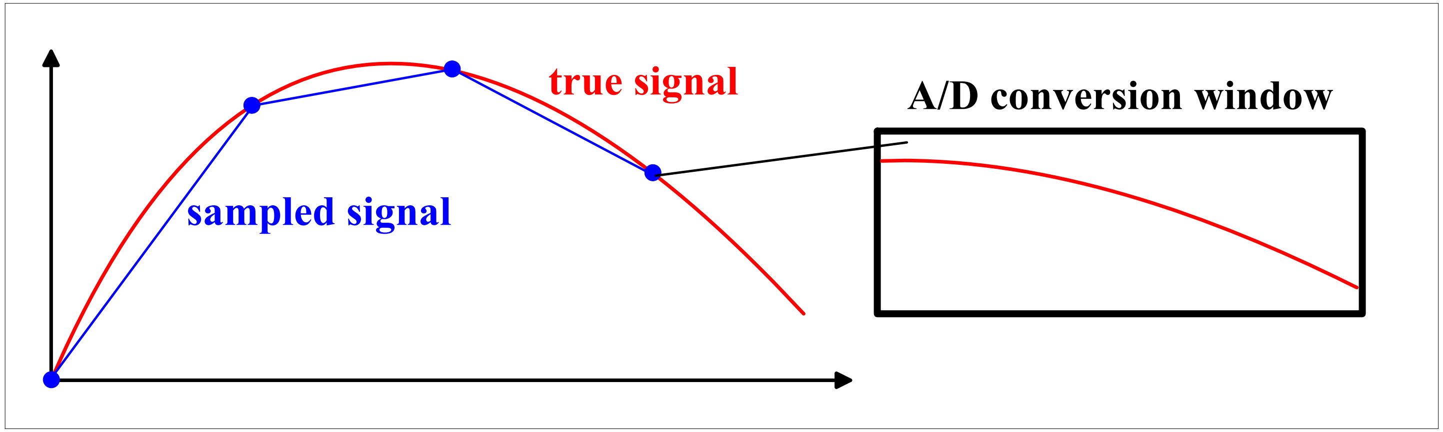
↓

spectral information

Digital information is always composed of basic information units which impose a certain granularity on the data. The finer the granularity, the better the results but the more information must be processed.

The finite data processing capabilities lead to a number of unwanted numerical effects which must be corrected by special filter techniques.

Jittering

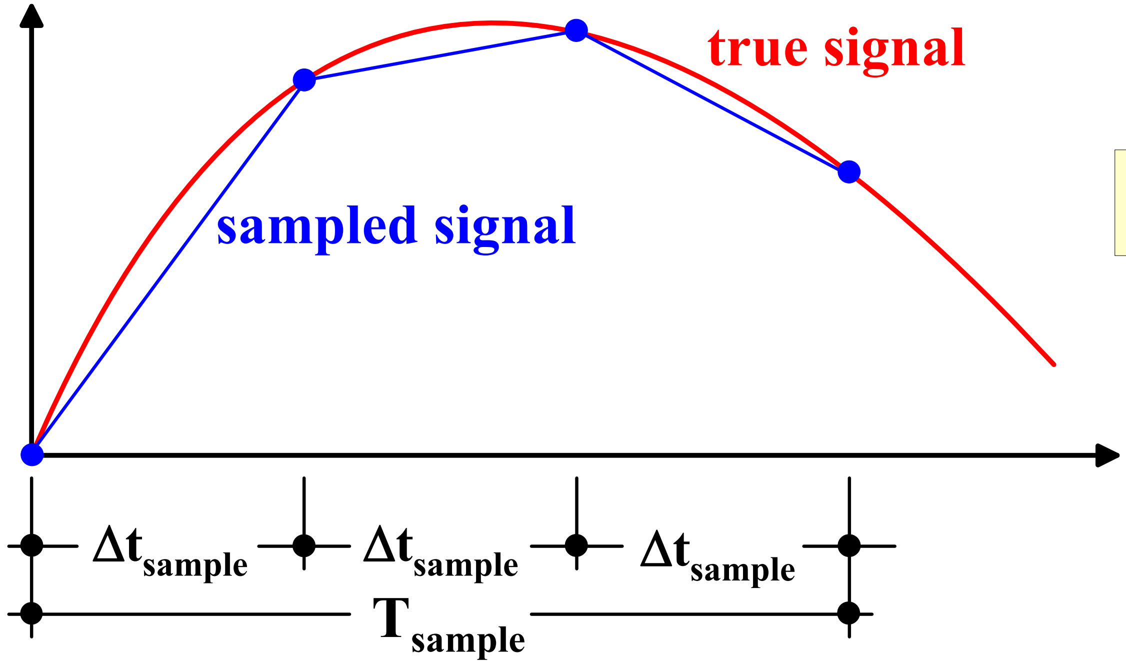


- To digitize the signal a short window is opened and the signal is sampled.
- During the sampling process the signal does not stay constant within the window.
- The mean value is taken as sampling value.
- This value is not necessarily to be found at the mid-point of the window.

In reality the signal is not sampled at perfectly equidistant time instance: *Jitter error*.

Sampling Rates in TD and FD

A time signal is sampled with a certain *sampling frequency*:



$$f_{sample} = 1/\Delta t_{sample}$$

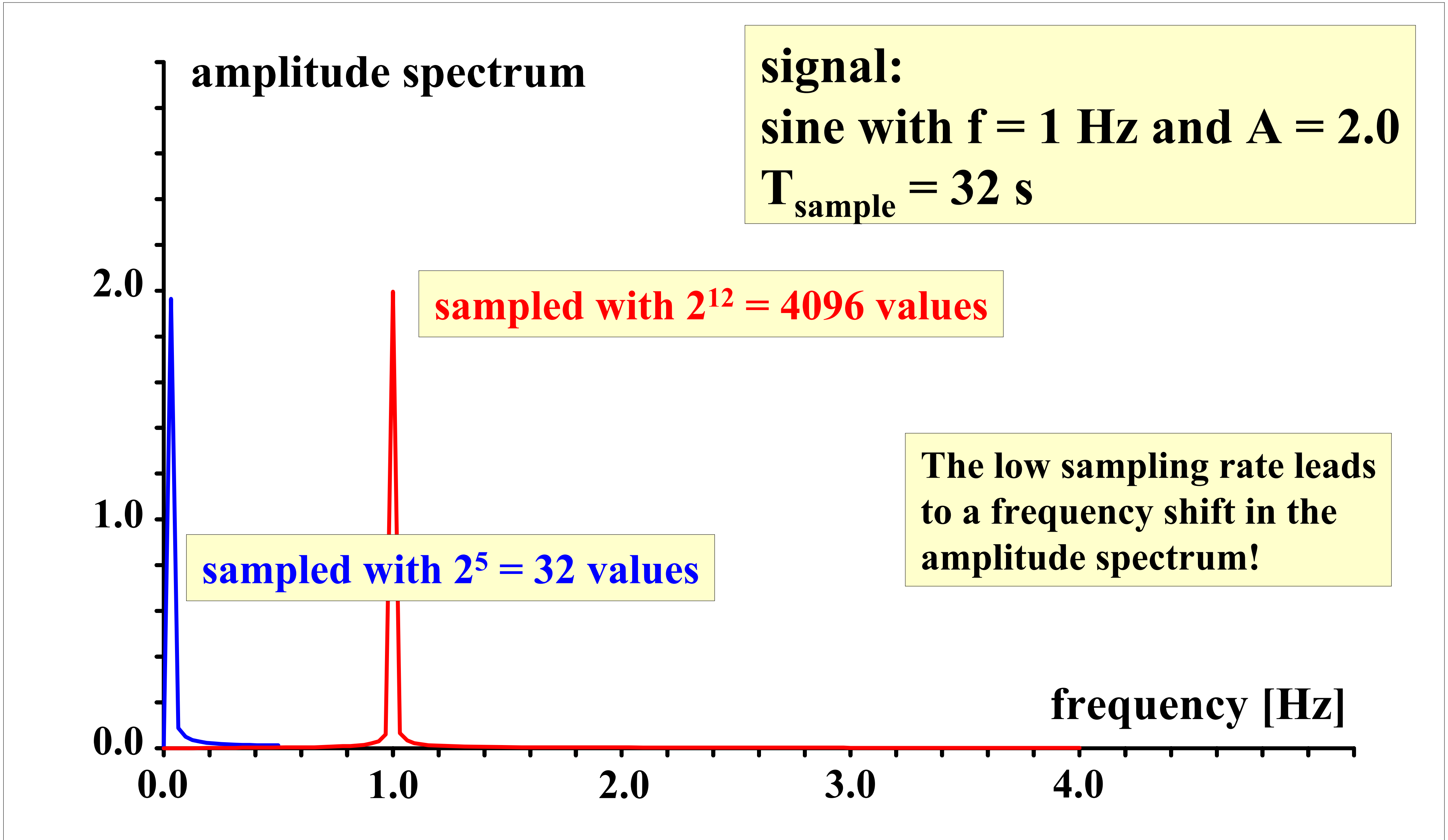
broad time domain → fine frequency resolution

$$\Delta f_{FD} = \frac{1}{T_{sample}}$$

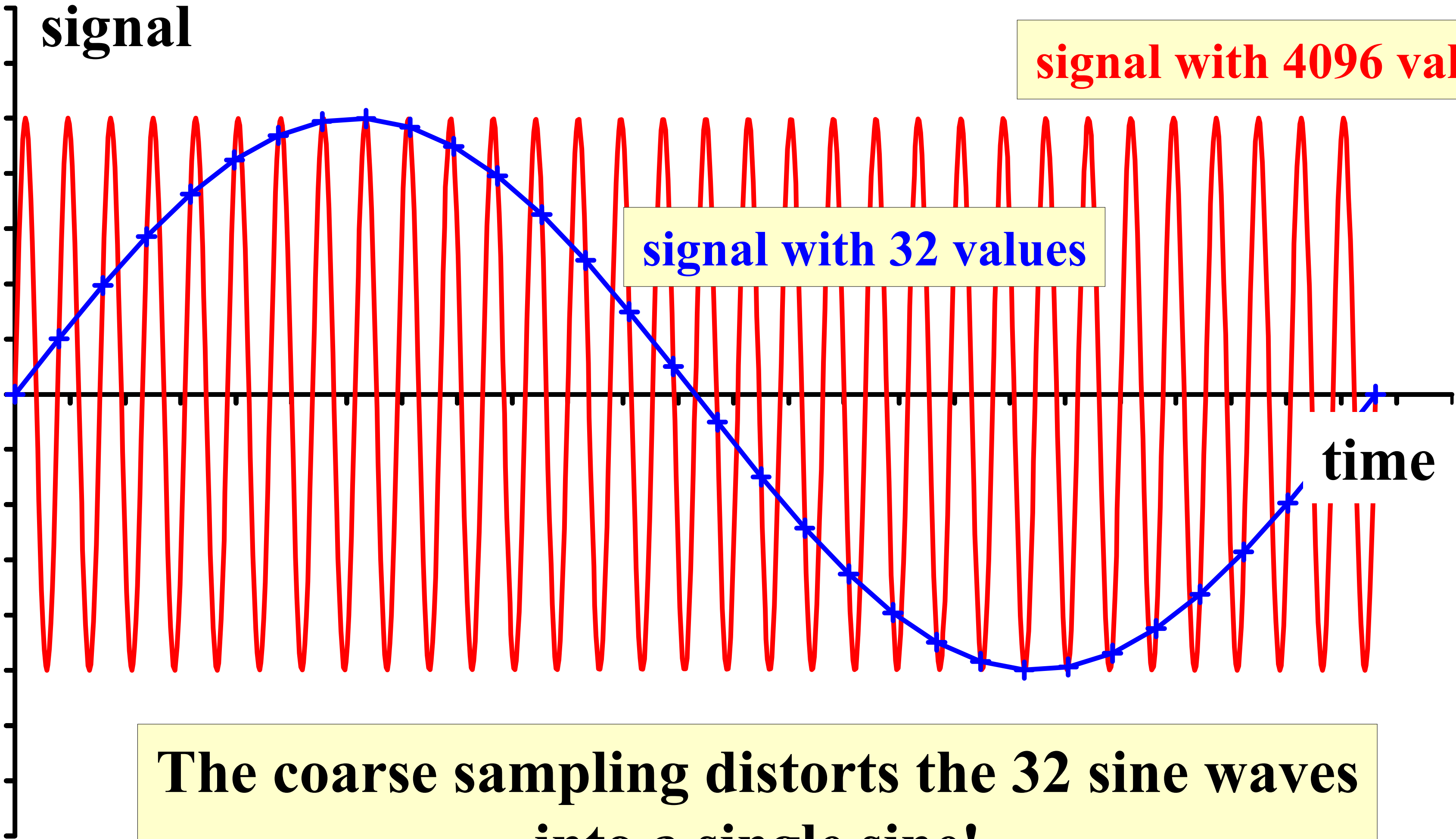
small time step → broad frequency domain

$$f_{max,FD} = \frac{1}{2\Delta t_{sample}}$$

Sampling Frequency too Low



Reason for the Frequency Shift

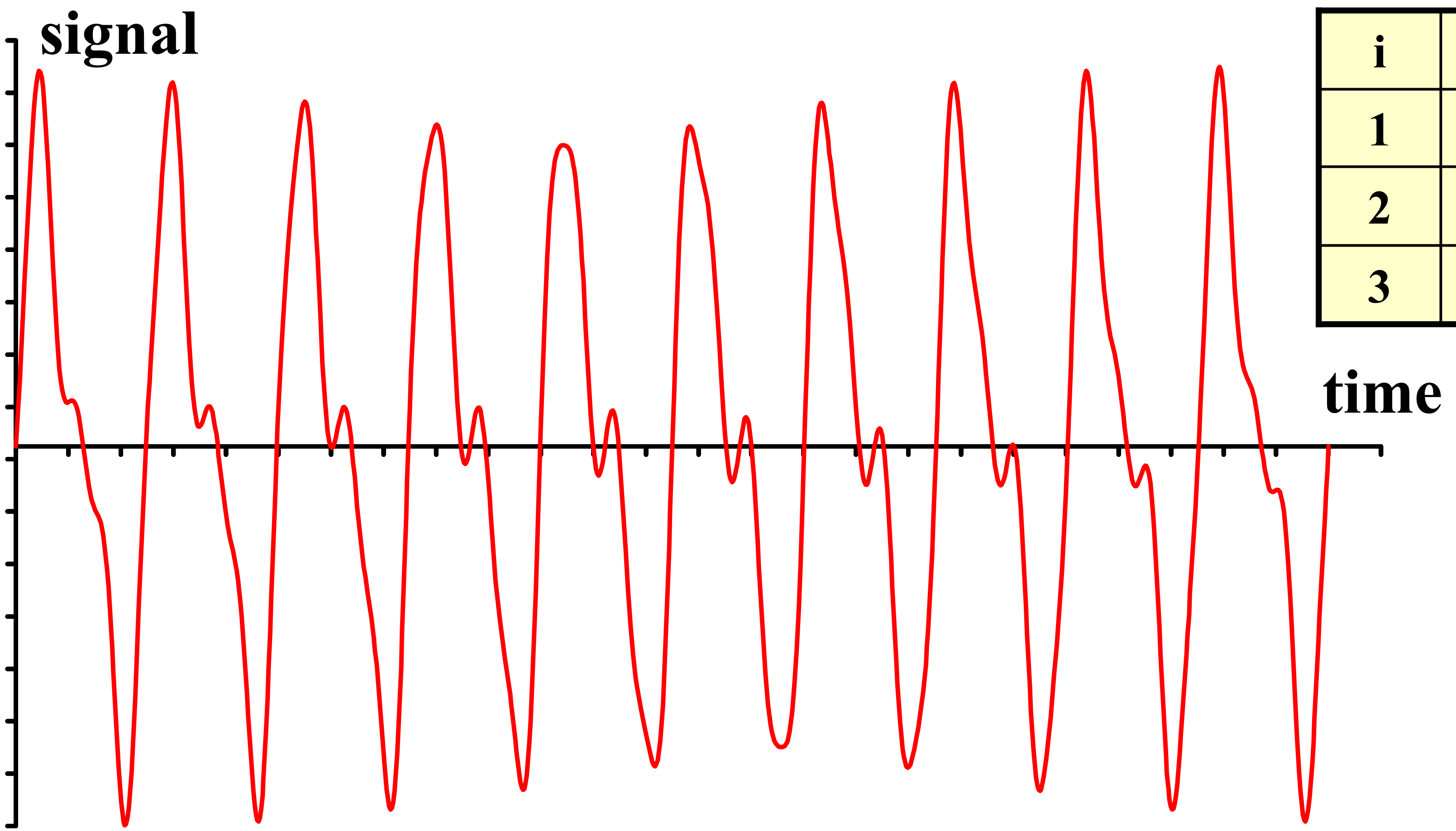


The coarse sampling distorts the 32 sine waves into a single sine!

Signal with one High-Frequency Part

$$v(t) = \sum A_i \sin \Omega_i t$$

i	f	A
1	1.0	2.0
2	2.0	1.0
3	98.4	0.3



Fast Fourier Transformation FFT

Assumption:

For our problem only frequencies of up to 4.0 Hz are relevant!

Chosen parameters:

$T_{sample} = 10.0\ s$

$N_{sample} = 2^{10} = 1024$

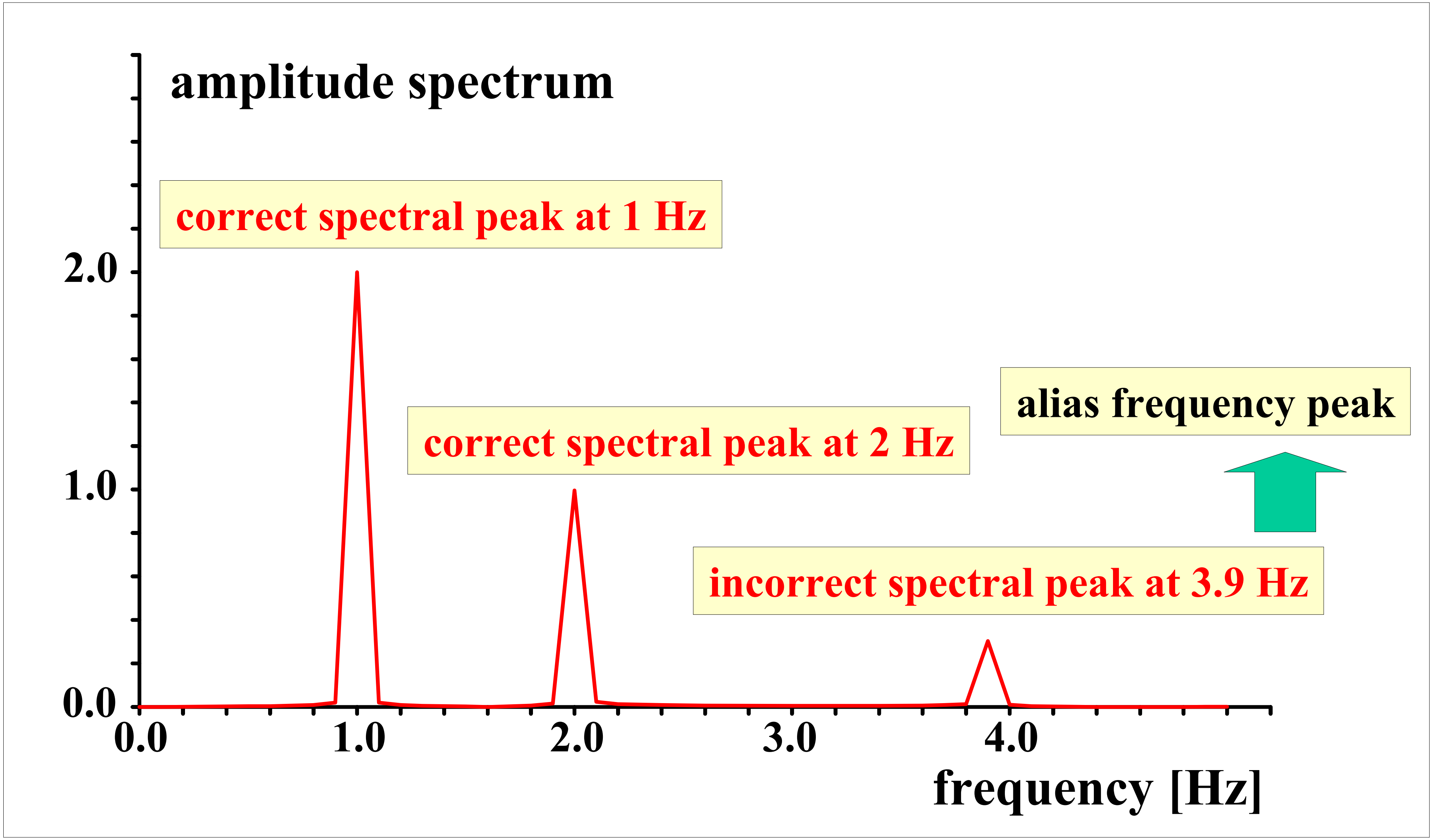
Calculated parameters:

$f_{max,FD} = 51.2\ Hz$

$\Delta f_{FD} = 0.009766\ Hz$

We expect a finely resolved amplitude spectrum with 2 peaks at the frequencies 1.0 Hz and 2.0 Hz.

Resulting Amplitude Spectrum



Aliasing

Aliasing effect:

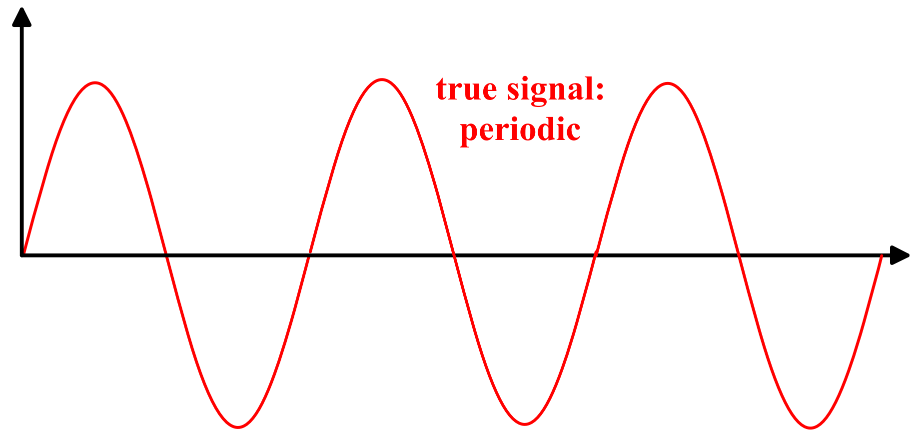
Spectral contributions from high-frequency oscillations which in reality lie outside the relevant spectral range are mirrored erroneously into the computed range. The mirroring occurs with respect to multiples of the NYQUIST frequency.
These fictitious peaks cannot be differentiated from the true peaks.

Anti-Aliasing filters:

Aliasing must not occur! All frequencies beyond a certain frequency bound are filtered out of the spectrum.

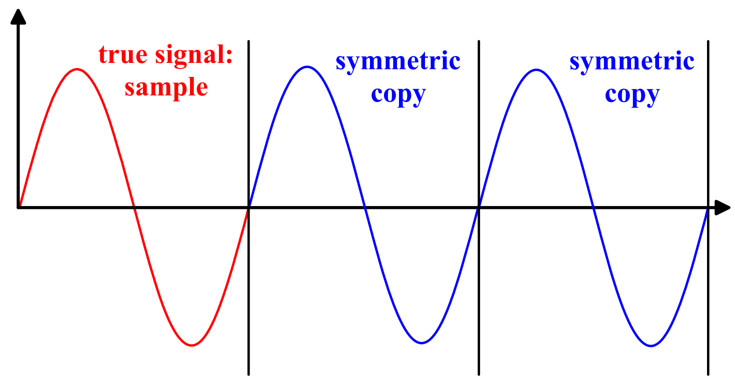
- **analog anti-aliasing filters: hardware**
- **digital anti-aliasing filters: software**

Periodicity of the FFT

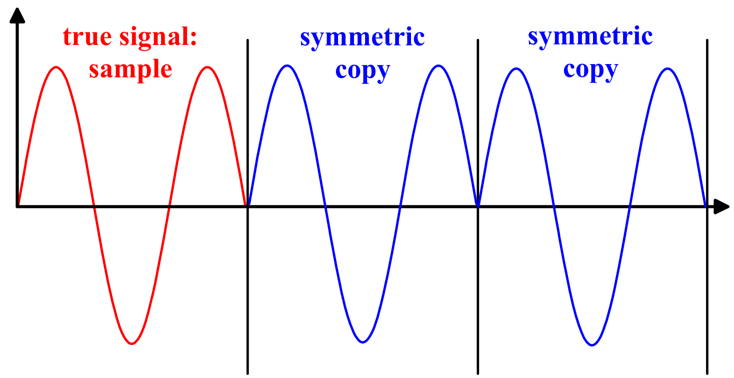


The FFT algorithm works as if the signal were periodic with the sample length T_{sample} as period. Even if the true signal is periodic, the periodically extended signal can differ from the true signal

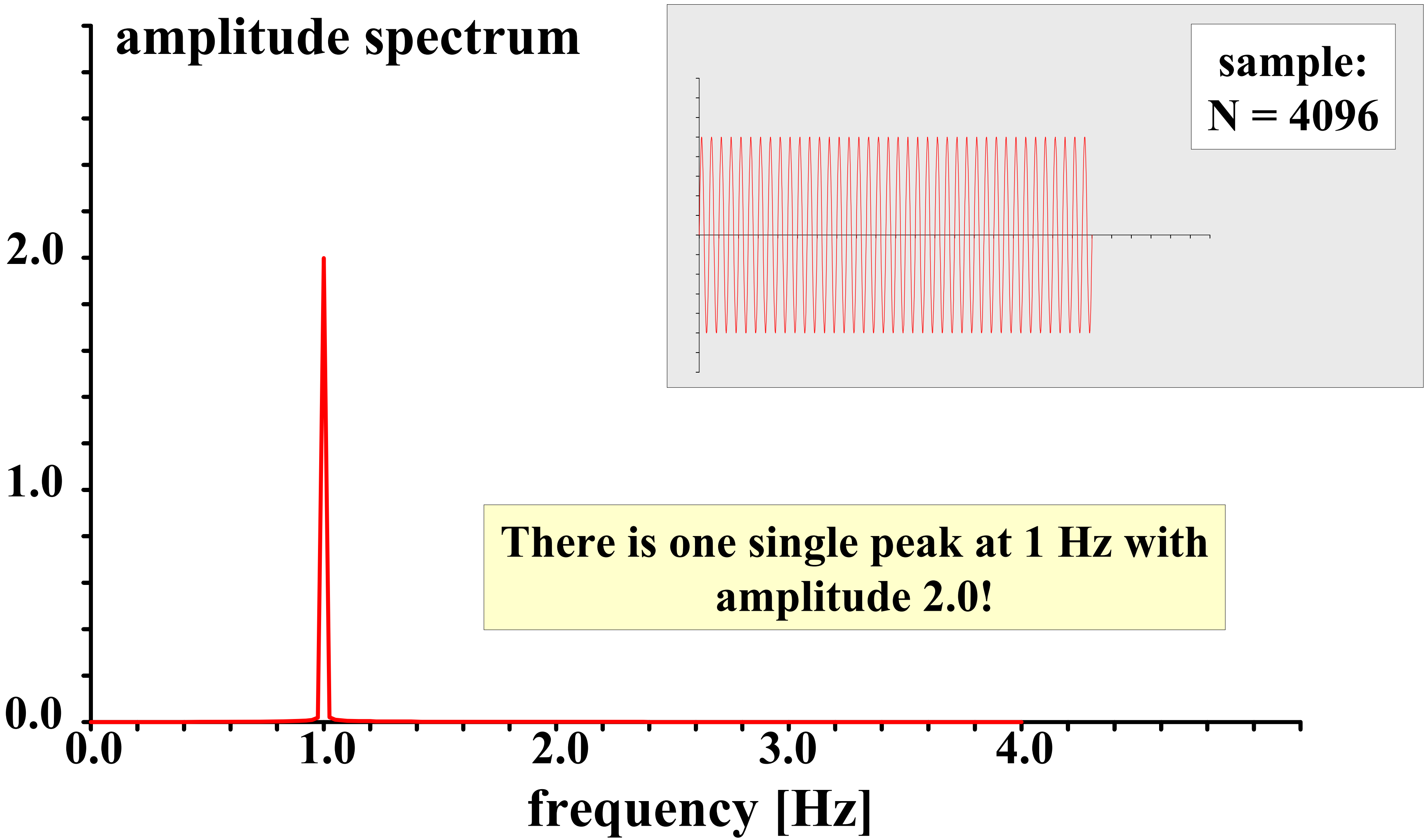
sample ends with an entire period:
continuous symmetric extension



sample ends with a truncated period:
discontinuous symmetric extension

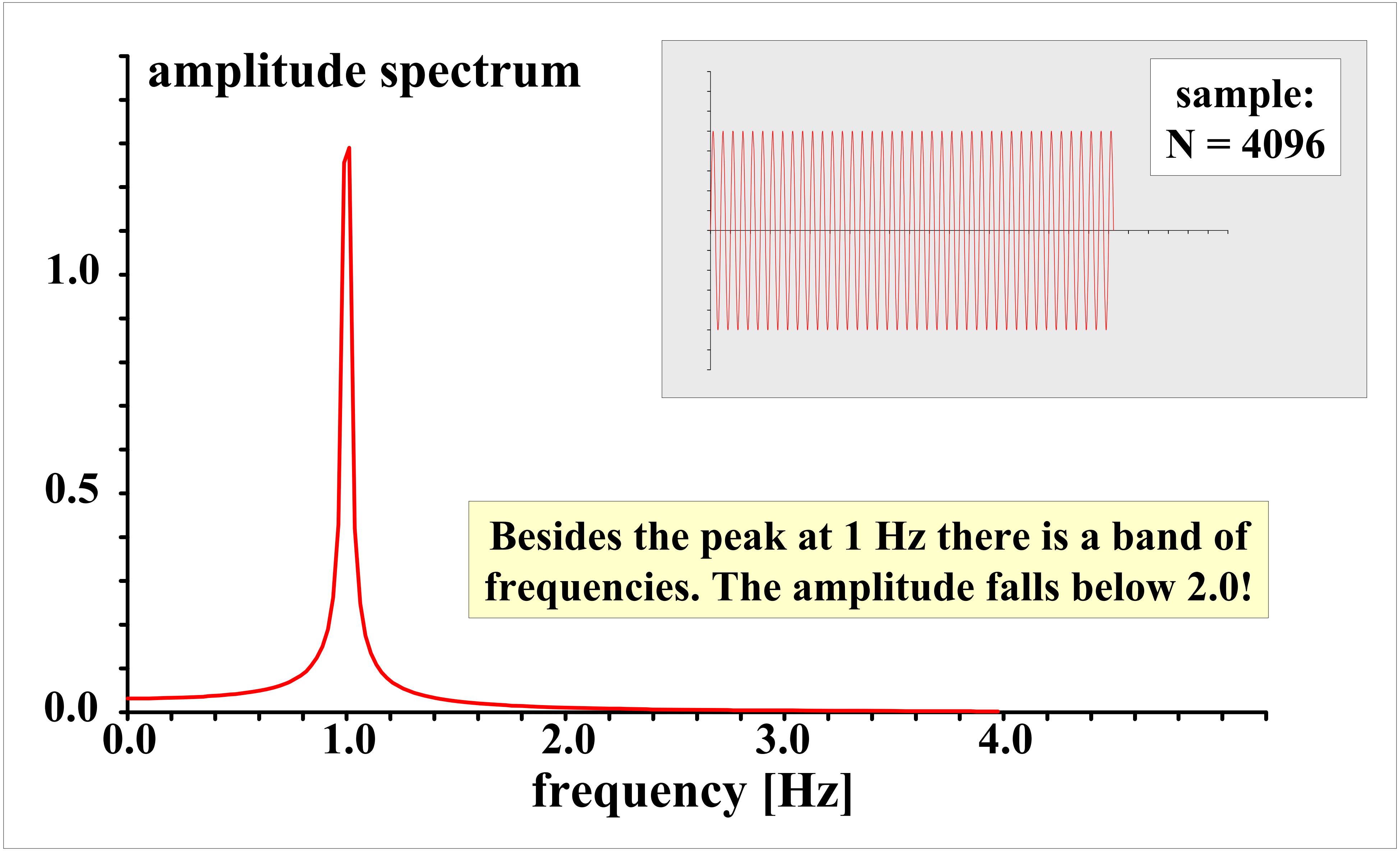


Sample 1: No Discontinuities



There is one single peak at 1 Hz with amplitude 2.0!

Sample 2: Discontinuous Slope



Leakage Effect

Leakage effect:

Due to discontinuities at the end of the sample frequencies are activated which in reality are not present in the sample (The peak leaks into its surroundings). They can obscure relevant frequency peaks in the neighbourhood of a large peak.

Weighting window technique:

The values within the sample window are multiplied with a special weighting function which imposes continuity between the sample and its periodic copy.

$$\tilde{v}(t) = v(t) \cdot h(t)$$

h(t): weighting function

Weighting Windows

There are many different weighting functions, e.g.:

- **general signals:**
  - **rectangular (uniform) window**
- **transient signals:**
  - **force window**
  - **exponential window**
  - **force-exponential window**
- **stationary signals:**
  - **Hanning (cosine-square) window**
  - **Hamming window**
  - **flattop window**
  - **Blackman-Harris window**
  - **Kaiser-Bessel window**
  - **.....**

Example: Hanning Window

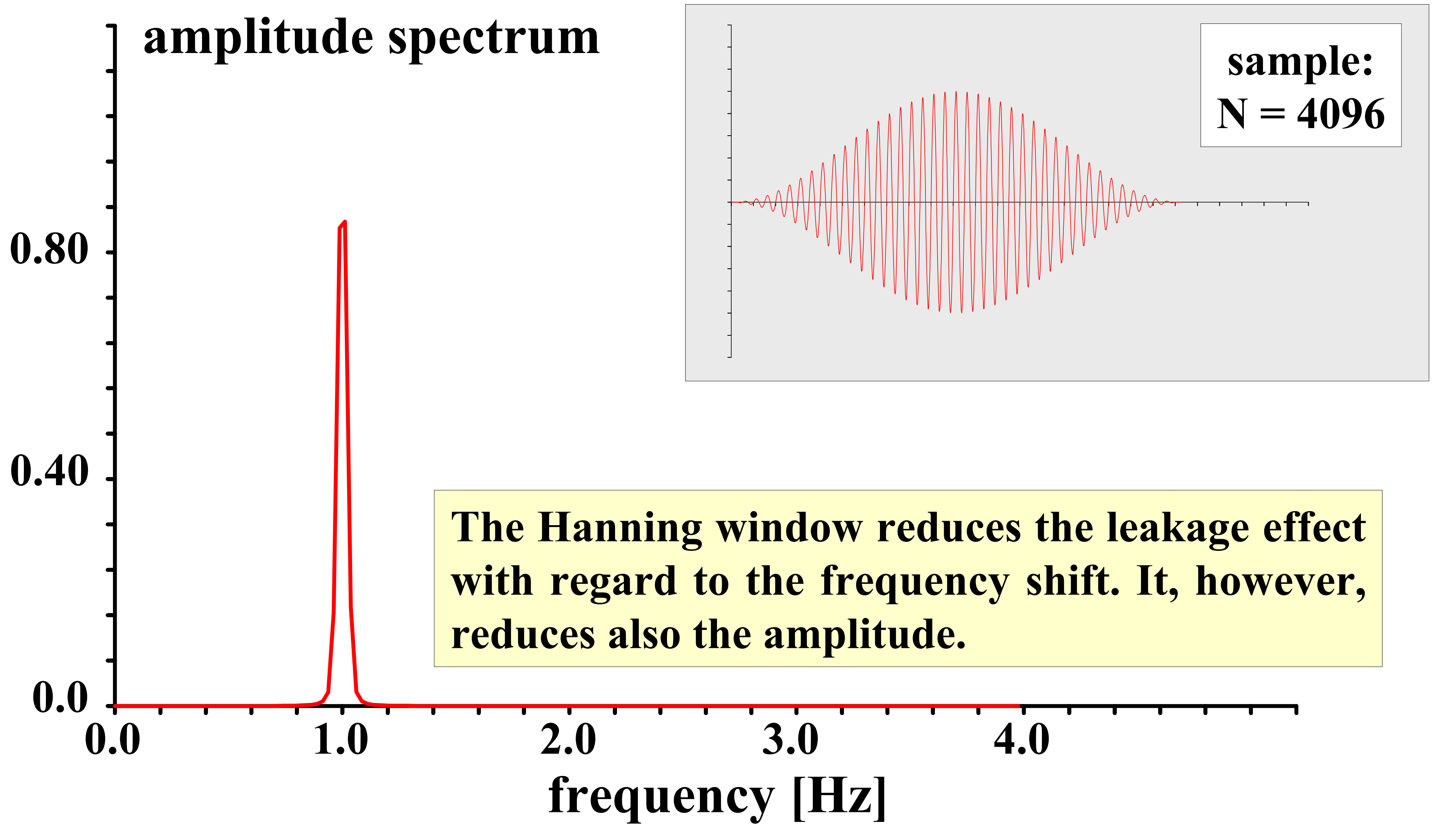
The weighting function $h(t^*)$ is described by a local time coordinate t^* within the sample which runs from –T/2 until T/2 ($T = T_{sample}$). It reads:

$$h(t^*) = \cos^2\left\{\frac{\pi t^*}{T}\right\}$$

It has the properties:

- **it is zero for $t^* = \pm T/2$,**
- **it is one for $t^* = 0$,**
- **the slope of $v(t) \cdot h(t)$ is zero for $t^* = \pm T/2$.**

Application of the Hanning Window



The Hanning window reduces the leakage effect with regard to the frequency shift. It, however, reduces also the amplitude.

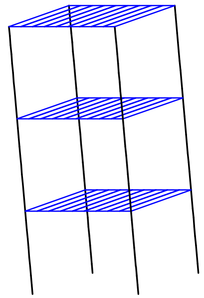
System Identification

true structure

analytical structural dynamics

excitation

FE-model



- **mode superposition**
- **direct integration**
- **FD method**

response

design

Problem:

How can be analyze the structure if its properties are (at least partly) unknown?

Review: FD Approach for SDOF-System

time domain:

$p(t)$

direct FT:

$$\underline{P}(i\Omega) = \int_{-\infty}^{+\infty} p(t)\, e^{-i\Omega t}\, dt$$

frequency domain:

$\underline{P}(i\Omega)$

transfer function:

$$\underline{V}(i\Omega) = \underline{H}(i\Omega)\, \underline{P}(i\Omega)$$

frequency domain:

$\underline{V}(i\Omega)$

inverse FT

$$v(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \underline{V}(i\Omega)\, e^{i\Omega t}\, d\Omega$$

time domain:

$v(t)$

Extension to General MDOF-Systems

FOURIER transform of the displacement vector (unknown)

Solution in the FD:

FOURIER transform of the load vector (given)

$$\begin{bmatrix} \underline{V_1} \\ \underline{V_2} \\ \bullet \\ \underline{V_2} \end{bmatrix} = \begin{bmatrix} \underline{H_{11}} & \underline{H_{12}} & \bullet & \underline{H_{1n}} \\ \underline{H_{21}} & \underline{H_{22}} & \bullet & \underline{H_{2n}} \\ \bullet & \bullet & \bullet & \bullet \\ \underline{H_{n1}} & \underline{H_{n2}} & \bullet & \underline{H_{nn}} \end{bmatrix} \begin{bmatrix} \underline{P_1} \\ \underline{P_2} \\ \bullet \\ \underline{P_2} \end{bmatrix}$$

matrix of transfer functions (structural properties)

Determination of the Transfer Matrix

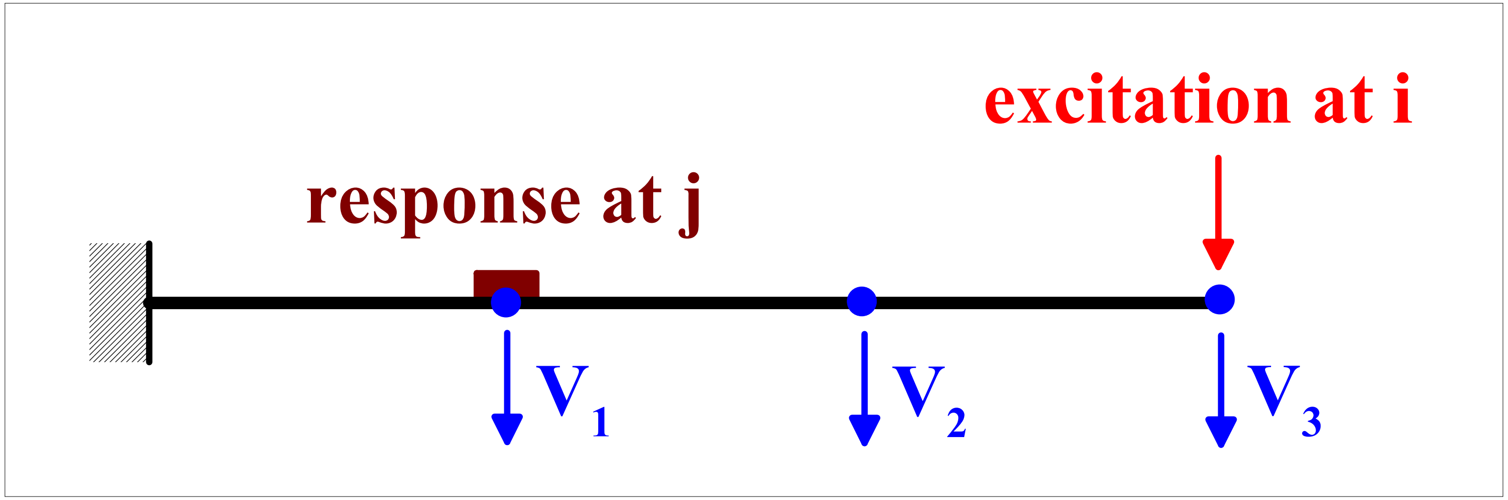
identification problem of calculating the transfer properties:

input: excitation
p_i measured

structure:

$$\left| H_{ji} \right| = \frac{\underline{U}_j}{\underline{P}_i}$$

output: response
u_j measured



Transfer Matrix via Power Spectra

Each measurement is imprecise due to measurement errors. Results can be improved by taking the mean of several records.

Mean values can be applied to power spectra but not to FOURIER transforms.

Compute cross-spectra with excitation:

$$\underline{S}_{u_j p_i} = \frac{1}{T} \underline{U}_j \underline{\tilde{P}}_i$$

$$S_{p_i p_i} = \frac{1}{T} \underline{P}_i \underline{\tilde{P}}_i$$

$$\underline{H}_{ji} = \frac{\sum \underline{U}_j \underline{\tilde{P}}_i}{\sum \underline{P}_i \underline{\tilde{P}}_i}$$

Final Remarks on System Identification

The direct measuring of all transfer functions for an MDOF system is not feasible for complex problems.

↓

A whole theory has been developed to perform system identification for arbitrary complex systems.

↓

Specialized field for specialists.

↓

We stop here and do not pursue this topic any further.