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Tuned Mass Dampers in the Design of Light-Weight Structures

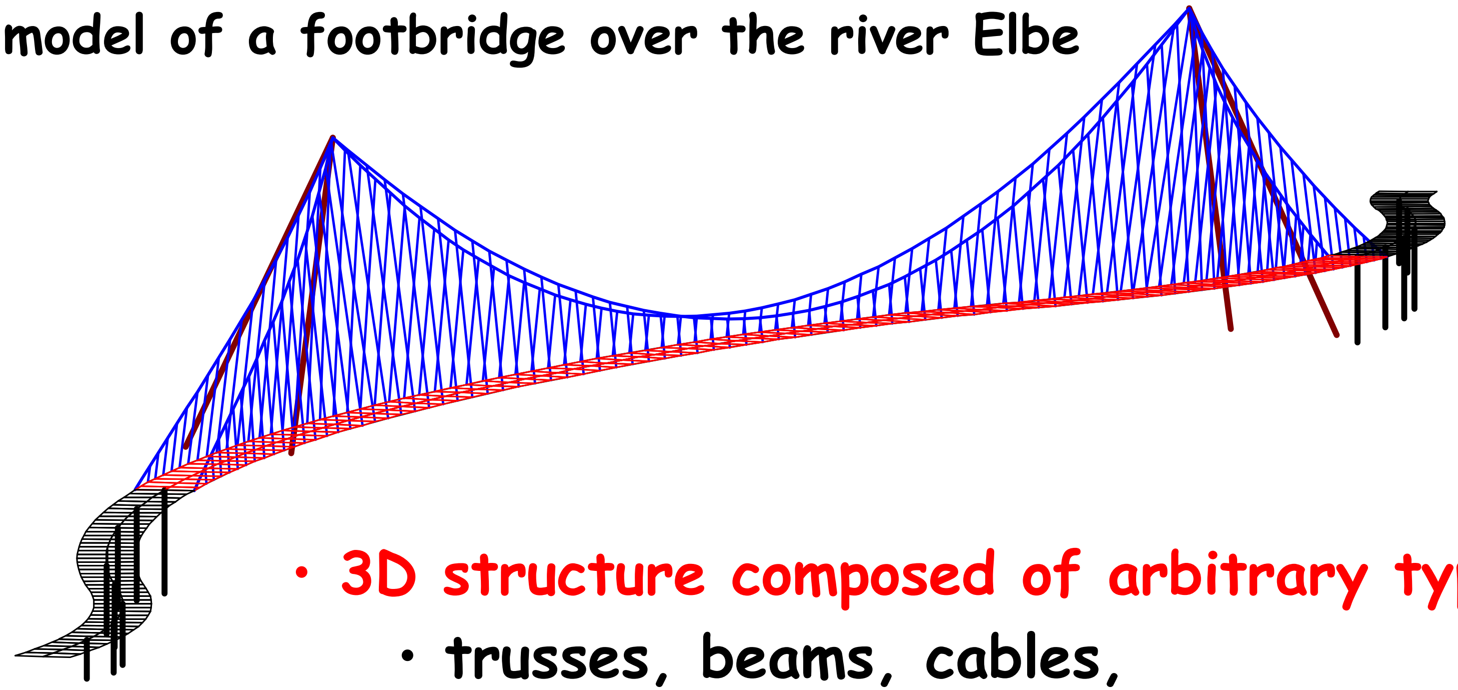
Theory, Numerical Simulation, Application

Lecture 04
Numerical Simulation

Structural Mechanics and Numerical Methods
Bergische Universität Wuppertal

Multi-Degree-Of-Freedom Models

FE-model of a footbridge over the river Elbe



- 3D structure composed of arbitrary types of members:
  - trusses, beams, cables,
  - slabs, shells, ...
- nonlinear behaviour can be modelled:
  - large deformations,
  - material nonlinearities: cracking of R/C, plasticity
  - aeroelastic effects, ...

Discrete Equation of Motion

linear case:

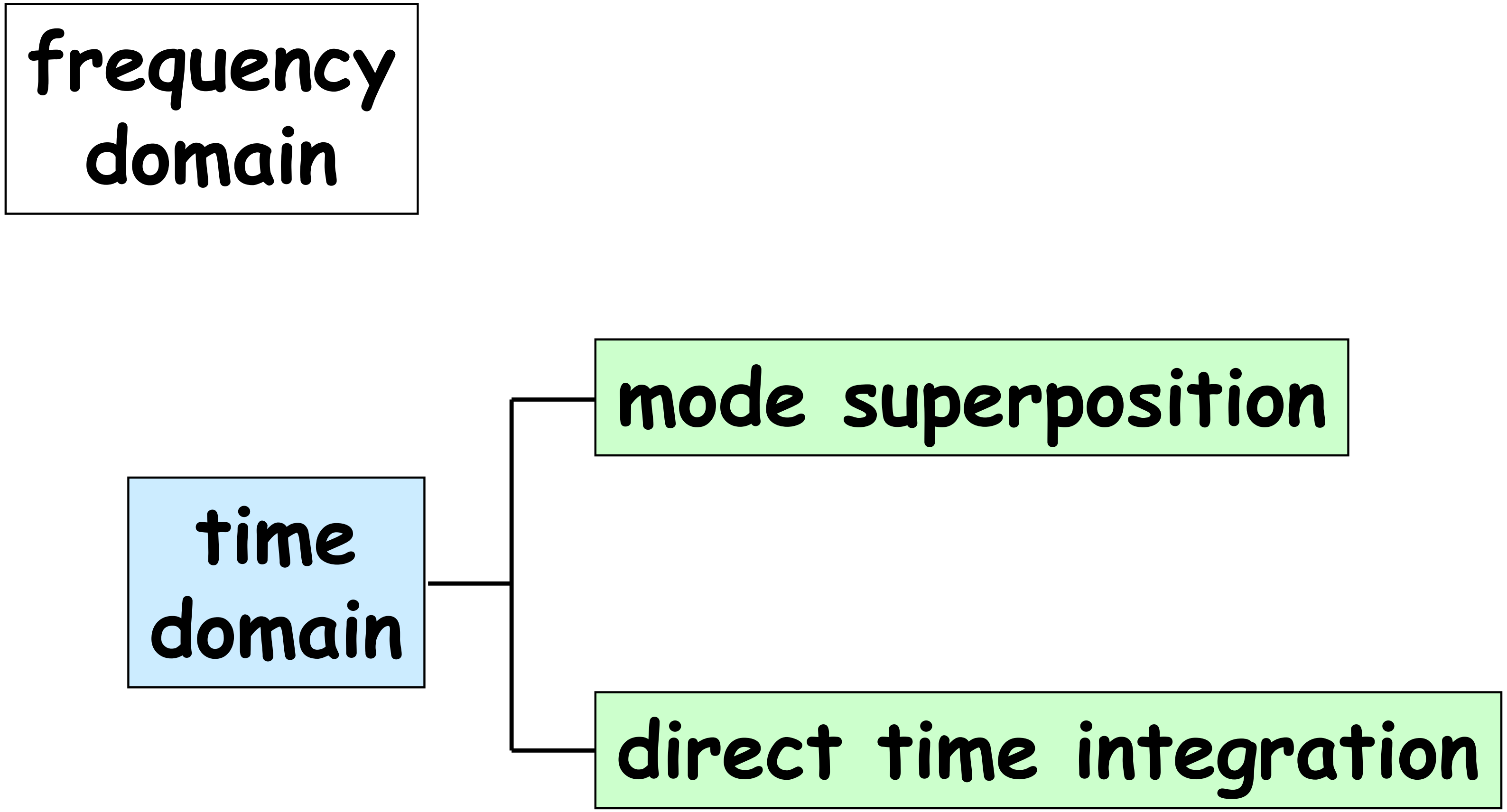
$$\mathbf{M}\,\ddot{\mathbf{V}} + \mathbf{C}\,\dot{\mathbf{V}} + \mathbf{K}\,\mathbf{V} = \mathbf{P}(t)$$

nonlinear case:

$$\mathbf{M}\,\Delta\ddot{\mathbf{V}} + \mathbf{C}_T\,\Delta\dot{\mathbf{V}} + \mathbf{K}_T\,\Delta\mathbf{V} = \mathbf{P}(t) - \mathbf{F}_m - \mathbf{F}_c - \mathbf{F}_k$$

We need suitable algorithms which can compute the response at discrete points in time for a given time history P(t). In the nonlinear case additional equilibrium iterations must be performed in each time step.

Overview: Linear Problems



Mode Superposition Method (MSM)

flow chart

linear eigenfrequency analysis

$$(\mathbf{K} - \omega^2 \mathbf{M})\mathbf{\Phi} = \mathbf{0}$$

→

transform into modal space

$$\tilde{m}_i = \mathbf{\Phi}_i^T \mathbf{M} \mathbf{\Phi}_i, \quad \tilde{k}_i = \mathbf{\Phi}_i^T \mathbf{K} \mathbf{\Phi}_i$$

$$\tilde{p}_i = \mathbf{\Phi}_i^T \mathbf{P}, \quad \tilde{c}_i = 2\xi_i \tilde{m}_i \omega_i$$

↓

solve modal equations of motion

$$\tilde{m}_i \ddot{\eta}_i + 2\xi_i \tilde{m}_i \omega_i \dot{\eta}_i + \tilde{k}_i \eta_i = \tilde{p}_i$$

←

result in original space

$$\mathbf{v}(t) = \mathbf{\Phi} \eta(t)$$

The modal decomposition method reduces the problem of n coupled equations to a problem of n independent SDOF problems (modal one-degree-of-freedom oscillators) which can be solved easily. The total solution is then the superposition of the n individual modal solutions.

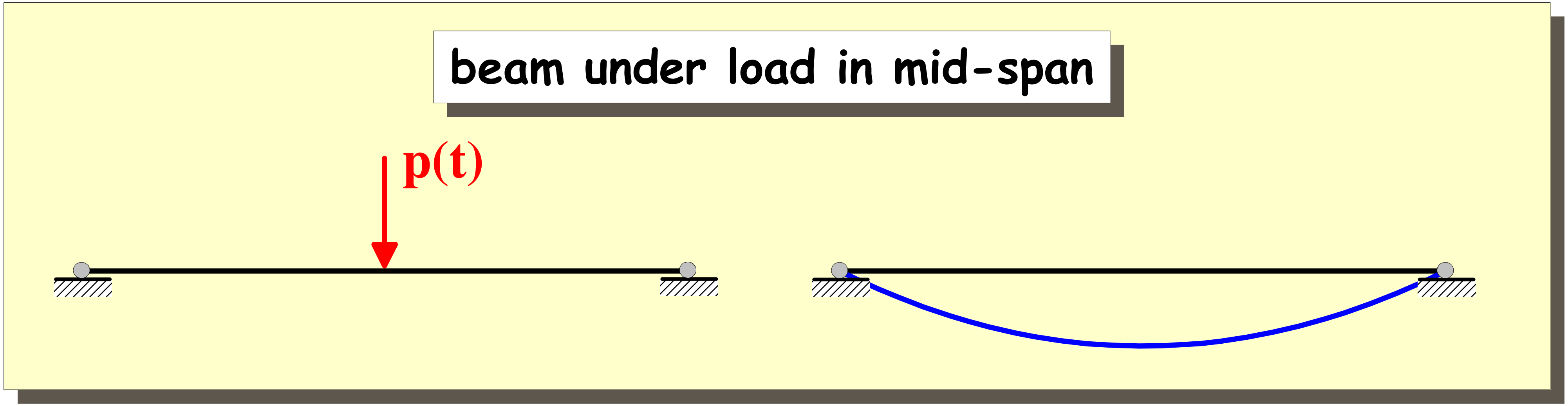
Reduction of the Number of DOFs

Original space:
The number of dofs is given by the discretisation. There can be thousands of dofs.

Structural behavior:
The displacement state can be synthesized as a superposition of all mode shapes. Higher mode shapes contribute less and less to the total response.

Modal space:
The number of dofs is given by the number of relevant mode shapes. Their number is only a fraction of the number of system dofs. Only some dozens of modes are needed.

Example for DOF Reduction



Only the first vibration mode is excited by the load:

$$w(x,t) = \eta_1(t)\,\Phi_1(x)$$

In modal space this problem has only one degree of freedom! The computing time is very short!

Direct Time Integration (DTI)

step-wise computation

start 0 → step 0 → step 1 → step 2 → step 3 →

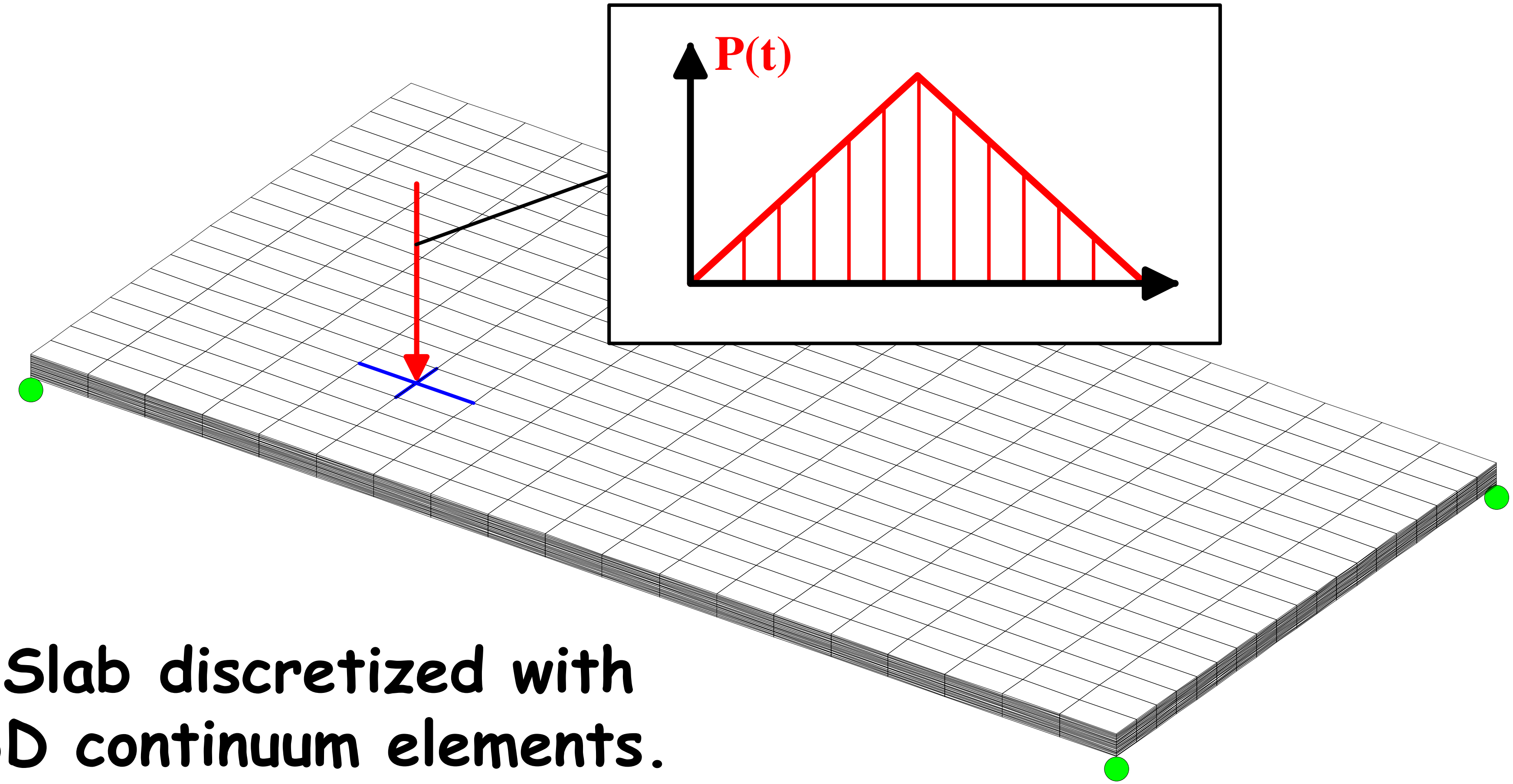
n-step procedure (explicit or implicit):

$$\mathbf{K}_{\text{eff}}\,\mathbf{V}^{(i)} = \mathbf{P}_{\text{eff}}^{(i)} = \mathbf{P}^{(i)} - \mathbf{F}^{(i-1)} - \mathbf{F}^{(i-2)} - \mathbf{F}^{(i-2)} - \ldots$$

$$\mathbf{F}^{(i-j)} = \mathbf{F}(\mathbf{V}^{(i-j)}, \dot{\mathbf{V}}^{(i-j)}, \ddot{\mathbf{V}}^{(i-j)})$$

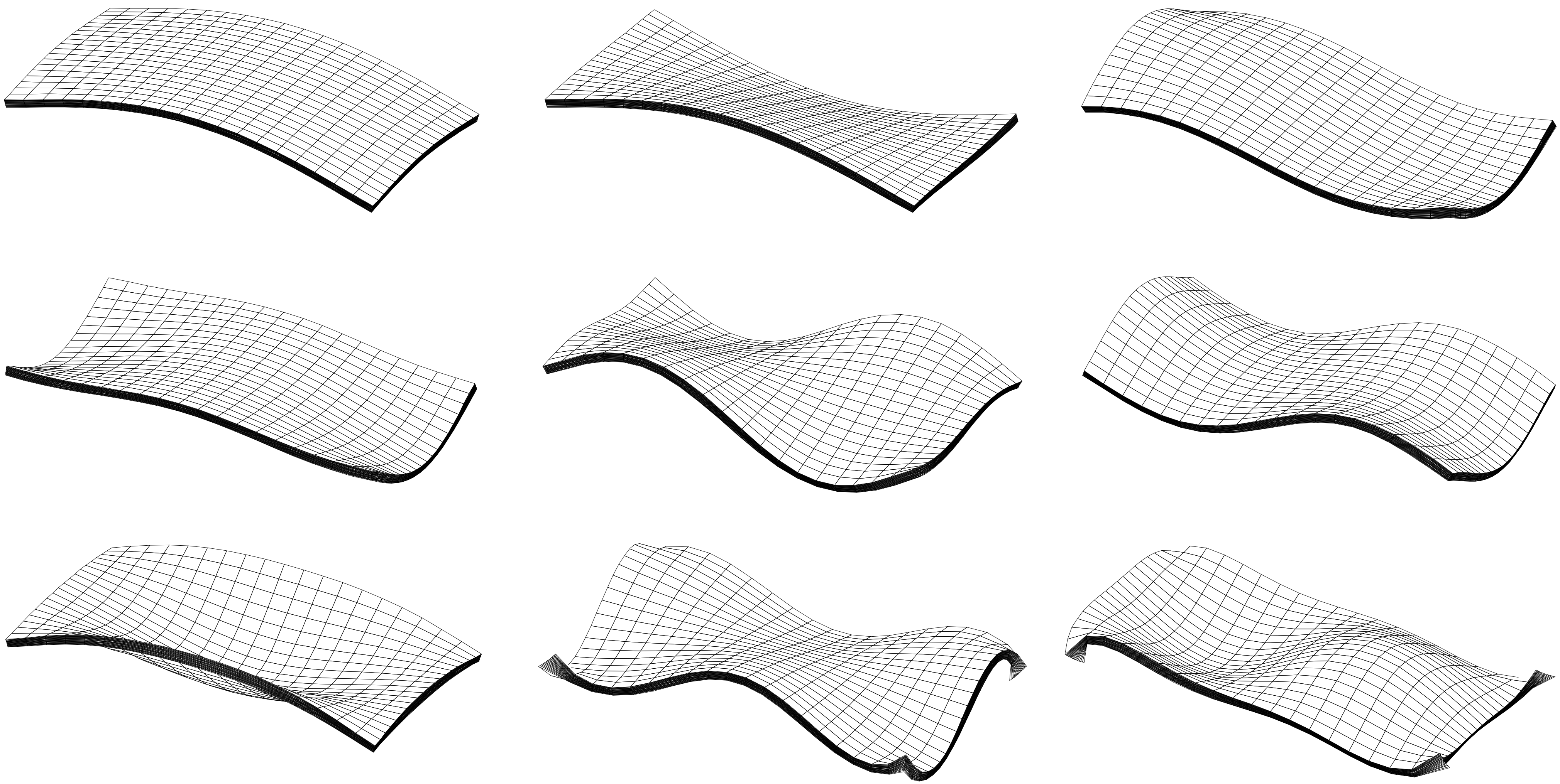
There are many time-stepping algorithms (Central Difference, Newmark, Wilson, ...) which differ in the formulation of K_{eff} and P_{eff}. DTI is numerically costly.

Example: MSM vs. DTI

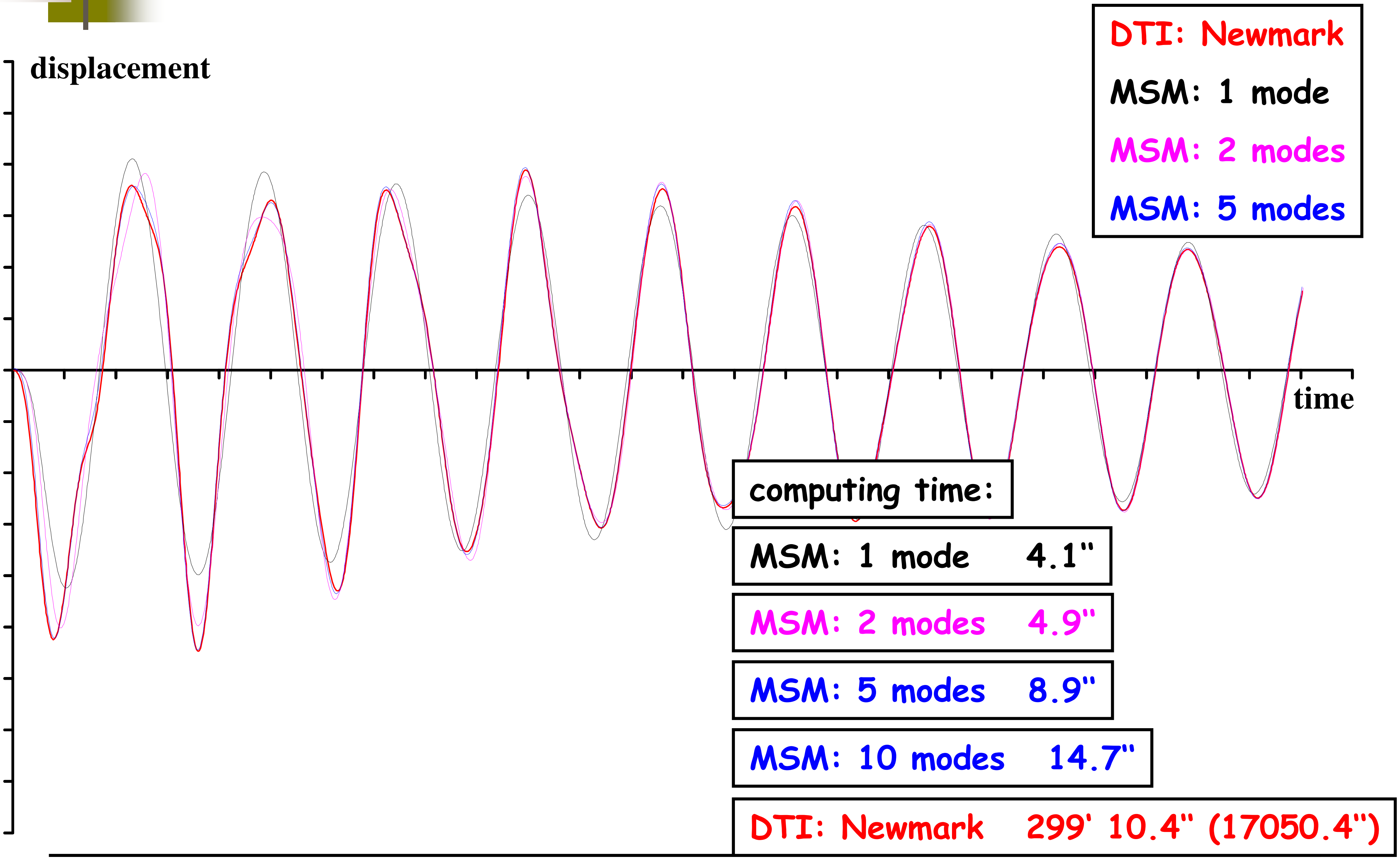


Slab discretized with 3D continuum elements. 23988 dof, large bandwidth.

MSM vs. TDI: Mode Shapes



Time Domain Response



computing time:

MSM: 1 mode	4.1"
MSM: 2 modes	4.9"
MSM: 5 modes	8.9"
MSM: 10 modes	14.7"
DTI: Newmark	299' 10.4" (17050.4")

Damping in MSM

There does not exist a damping matrix in the MSM!

Damping is defined for each mode shape separately (modal damping) as a percentage ζ of critical damping for the specific mode shape.

☺ **We have complete control over all mode shapes.**

☹ **This decoupled damping does not allow for discrete dampers, since a discrete damper would lead to a fully coupled damping matrix in modal space.**

Damping in DTI

There do not exist practically manageable damping models on the element level. Therefore no element damping matrices are assembled into a system damping matrix.

Overall damping is defined on the structural level as mass- and stiffness-proportional damping (so-called Rayleigh damping). Discrete dampers can be added!

☺ Discrete dampers can be captured.

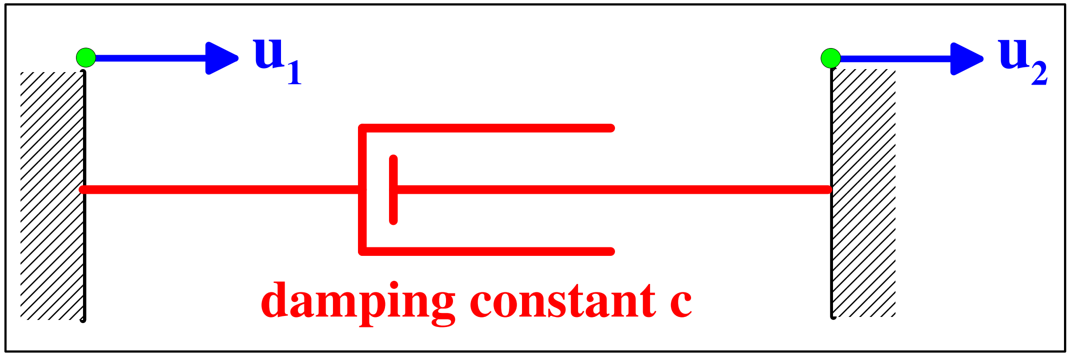
☹ We have only control over the damping of two mode shapes. The damping of the other mode shapes follows automatically and we must strike a compromise to account for more than two modes.

Discrete Dampers

For a discrete damper the damping constant c is explicitly known in [kNs/m] from the manufacturer.

viscous damping element (Gerb Engineering, Essen)

damping element with 2 dofs



$$\mathbf{C}_{\text{element,local}} = \begin{bmatrix} c & -c \\ -c & c \end{bmatrix}$$

Damping Matrix in DTI

total damping matrix

$$\mathbf{C}_{\text{tot}} = \mathbf{C}_{\text{struct}} + \mathbf{C}_{\text{element}}$$

structural damping via Rayleigh damping

$$\mathbf{C}_{\text{struct}} = \alpha_{\text{M}}\mathbf{M} + \alpha_{\text{K}}\mathbf{K}$$

The parameters α_M and α_K can be determined such that two damping ratios ξ_i and ξ_j can be prescribed for two arbitrarily chosen modes.

Algorithm for TMD-Simulation

The modeling of a structure with TMDs requires discrete dampers. Therefore *only direct time integration methods* are suitable!

Attention: FE-programs (e.g. SAP2000) allow us to build up a discretization with discrete dampers and then to perform an analysis with the modal superposition method. Without our noticing, however, the damping properties of the discrete dampers are automatically neglected, because the algorithm does not use a system damping matrix. Then we might (hopefully) wonder why our TMD is not working correctly.

We, e.g. the user, must pay attention! The programs themselves do not object to nonsensical usage.

Modal Masses

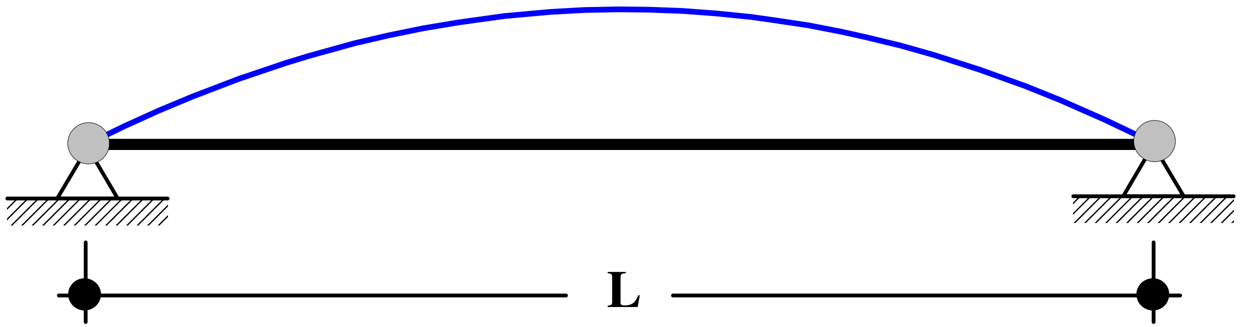
The mass of the TMD is chosen as a certain percentage (2 % - 5 %) of the ***modal mass***, not the true mass of the structure.

The modal masses are defined in such a way that the kinetic energy of the modal sdof-system is equal to the kinetic energy of the corresponding mode of the true system.

The stiffness of the modal sdof-system is then given by the requirement that its eigenfrequency must remain equal to the original eigenfrequency

Example: Beam Continuum

assumption: structure vibrates affine to the first mode of vibration



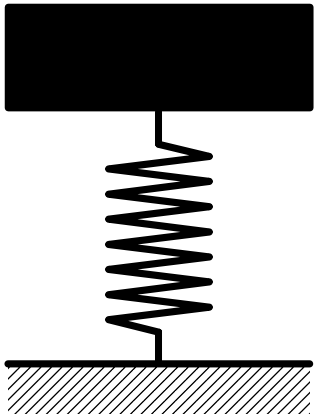
$$w(x,t) = \hat{W}\sin(\omega t)\sin(\pi x / L)$$

$$E_k = \int_L \frac{1}{2} m \dot{w}^2 dx = \frac{1}{2}\int_L m\hat{W}^2\omega^2\cos^2(\omega t)\sin^2(\pi x / L)dx$$

$$= \frac{1}{2} m\hat{W}^2\omega^2\cos^2(\omega t)\frac{1}{2}L = \frac{1}{2}\left\{\frac{1}{2}M_{tot}(\hat{W}\omega\cos\omega t)^2\right\}$$

Continuum ⇒ SDOF-System

assumption: SDOF-system vibrates with the same frequency



$$v(t) = \hat{V}\sin(\omega t)$$

$$E_k = \frac{1}{2}M\dot{w}^2 = \frac{1}{2}M\hat{V}^2\omega^2\cos^2(\omega t)$$

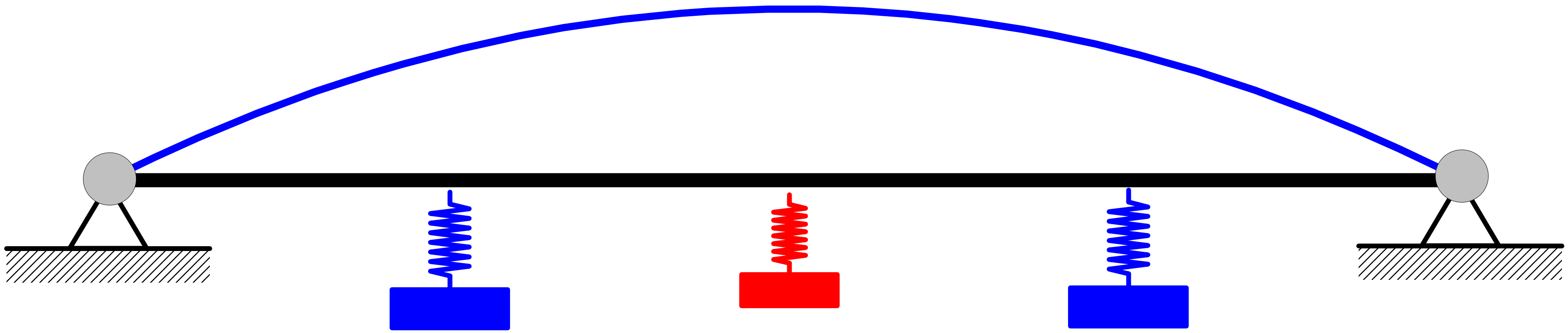
energies of SDOF-system and continuum must be identical:

$$\Rightarrow \quad M = \frac{1}{2}M_{tot}\frac{\hat{W}^2}{\hat{V}^2}$$

what is the significance of the amplitude ratio?

Location of the TMD

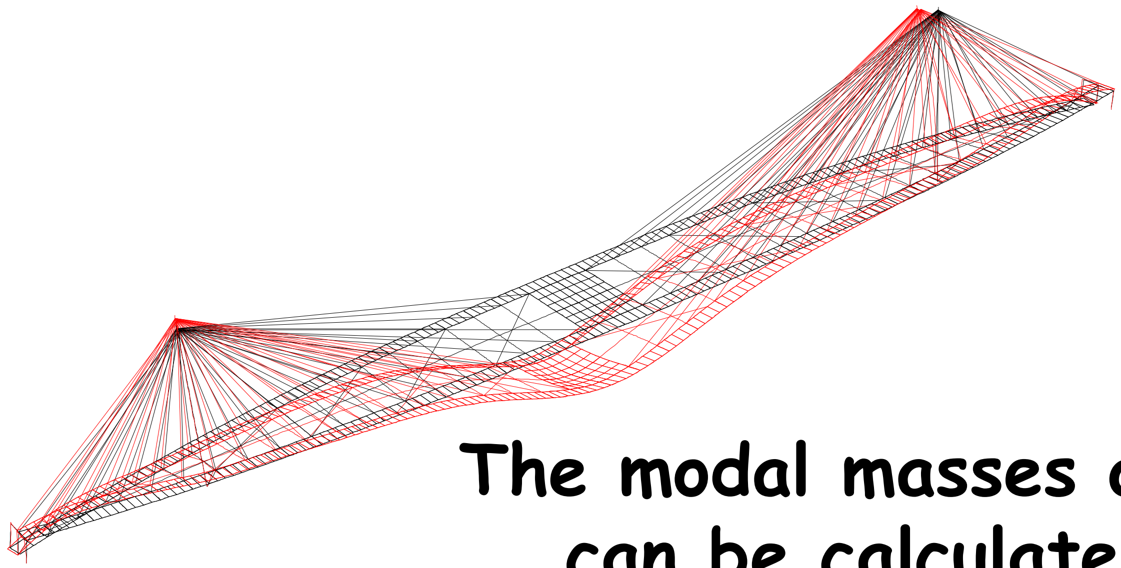
A modal SDOF-system has no physical location. It is just a purely mathematical abstraction. The TMD, however, is situated at some specific location within the structure. The amplitude factor is 1 if the TMD is put exactly where the corresponding mode has its largest amplitude. Then it is most effective. If it is put somewhere else, then its mass must be increased to obtain the same effect.



$M_{mod1} = 0.5M_{tot}$ $M_{mod} = 0.5M_{tot}$ $M_{mod2} = 0.5M_{tot}$

If two TMDs are put in quarter span, each one must be as massive as the one in mid-span!

Discrete Systems – FE-Problems

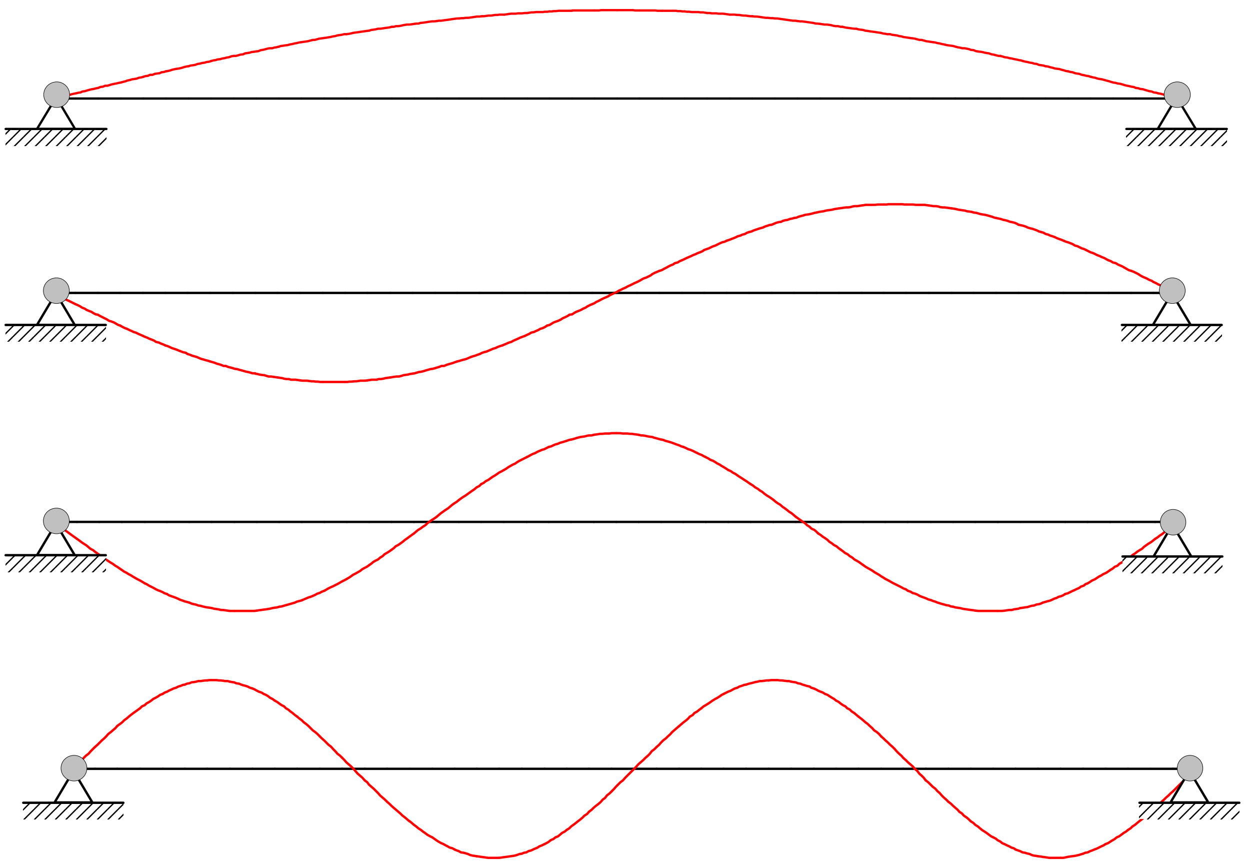


The modal masses of a discrete FE-model can be calculated from the discrete eigenmodes Φ and the mass matrix $\mathbf{M}$ according to:

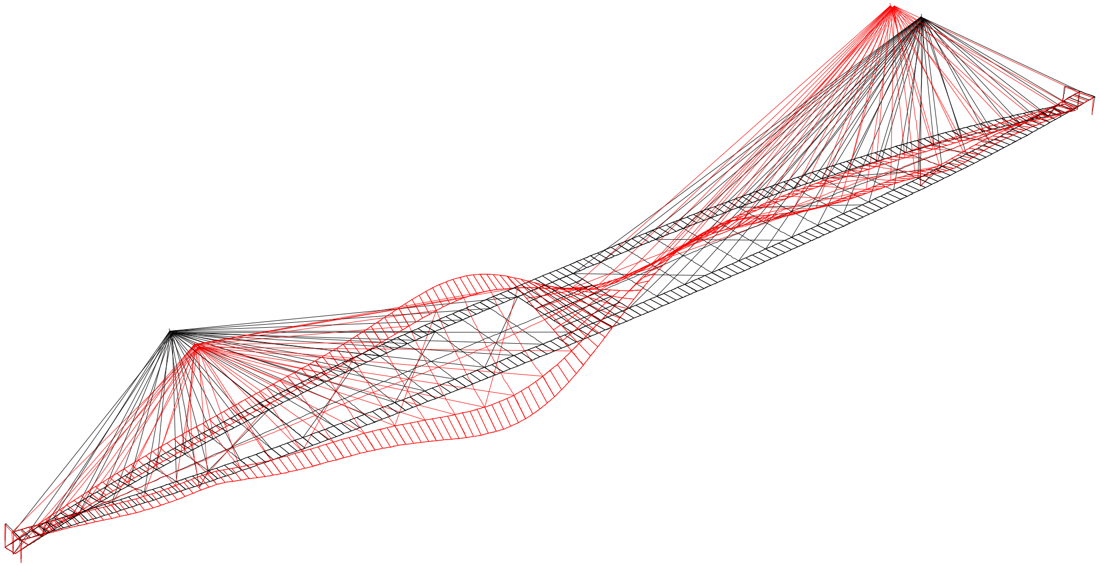
$$M_{mod} = \frac{1}{\Phi_{TMD}^{2}} \boldsymbol{\Phi}^{T} \mathbf{M} \boldsymbol{\Phi}$$

Multi-Mode Scenario

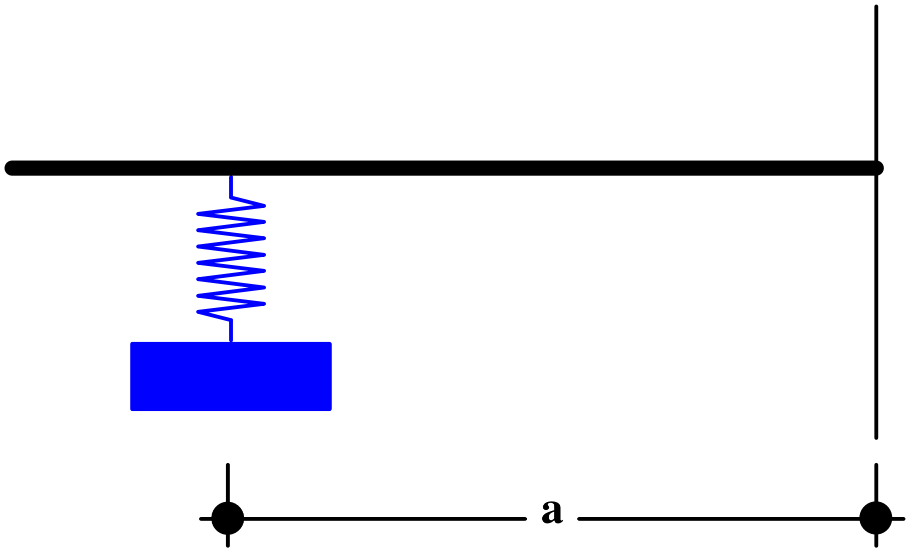
Each mode possesses its own different modal mass. So if several modes must be damped, then for each one the TMD-mass must be computed from the corresponding modal mass!



TMD for Torsion



In a 3D FE-Model there are no global torsional degrees of freedom, only displacements. For an equivalent model for a predominantly torsional mode, there must be calculated a corresponding moment of mass inertia from the modal mass.



$$\Theta = M_{mod} \cdot a^2$$

Outlook to Next Lecture

Some aspects of the numerical simulation of structures have been discussed. Many more remain ⇒ entire lecture series on structural dynamics.

Next there will be presented some practical applications where these techniques have been used in the design: footbridges, connecting walkways, bell towers.