Wolfhard Zahlten

Lecture Series:

Structural Dynamics

Lecture 3:

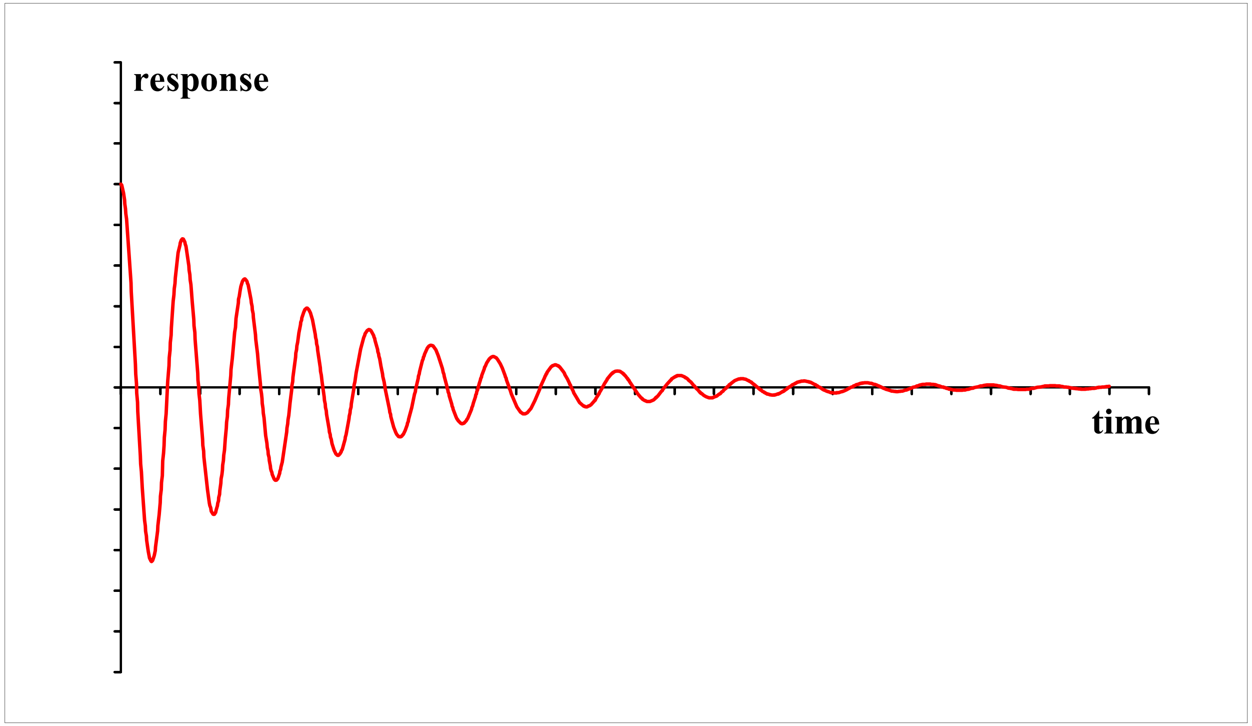
Structural Damping

Overview

- Phenomenological view
- Some damping models:
  - friction damping
  - hysteretic damping
  - viscous damping

Lecture 2 versus Experience

The free, undamped vibration of Lecture 2 would continue ad infinitum without ever coming to rest. Daily experience, however, tells us that, without external intervention, any system invariably comes to rest. We will later see that this effect is vital and must be included in our mathematical model.



Reason for the Damping Effect

The reason for damping in general is *energy dissipation:*

$$E_{tot} = E_{kin} + E_{pot} - E_{diss}$$

A part of the total „vibration energy“ $E_{kin} + E_{pot}$ is continually transformed into other forms of energy (heat, sound, ...) which leave the mechanical system (*nonconservative system*) and therefore does no longer contribute to the vibration. This effect is called *damping*.

Types of Damping

total damping

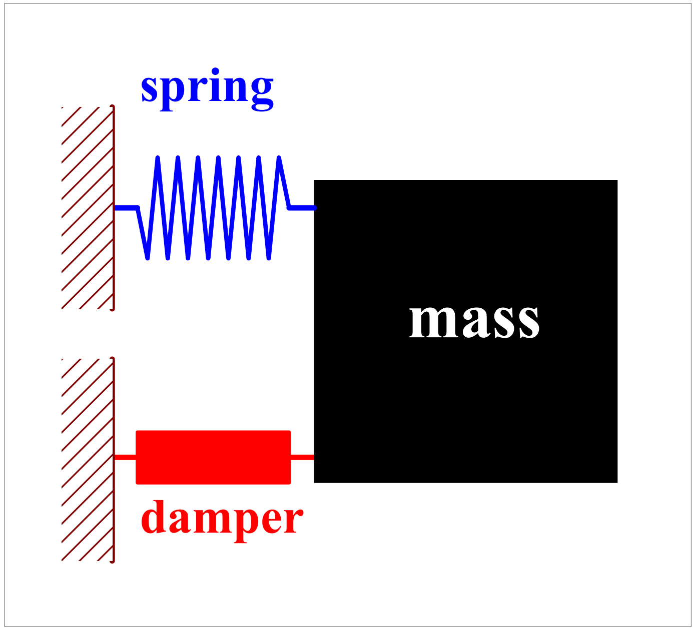
external damping:
- air resistance

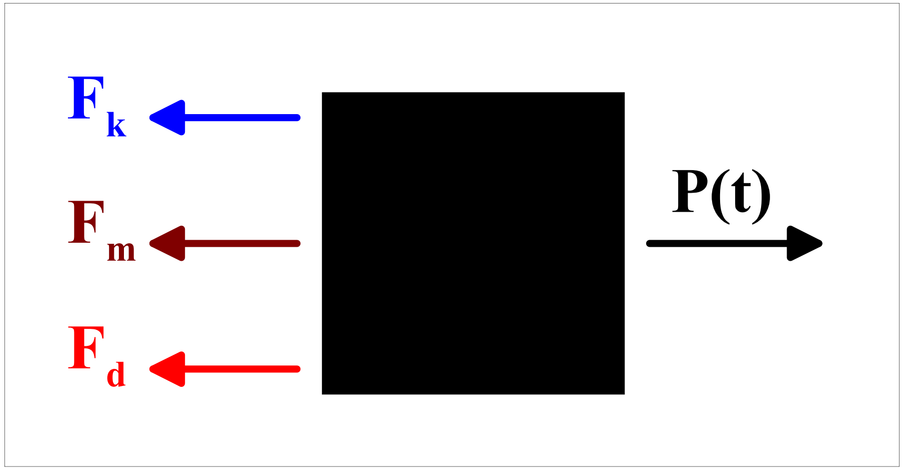
internal damping:
- material damping
- member damping
- soil damping
- support damping

Damping mechanisms are very complex and difficult to capture. Often the exact underlying reasons cannot be identified.

Heavily simplified models, often purely empirical.

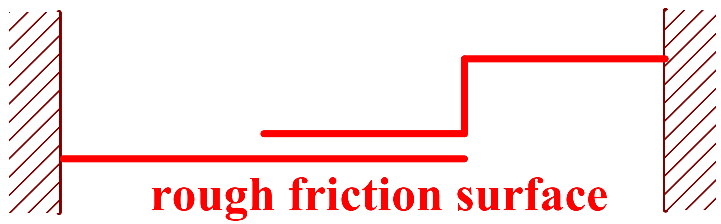
Damping Models: Generalities





- **inertial force:** $F_m = m\ddot{v}$
- **elastic force:** $F_k = kv$
- **damping force:** $F_d = ?$

Model 1: Friction Damping



The damping force acts in negative displacement direction:

$$F_d = -f\,\mathrm{sign}(\dot{v}) = -f\,\frac{\dot{v}}{|\dot{v}|}$$

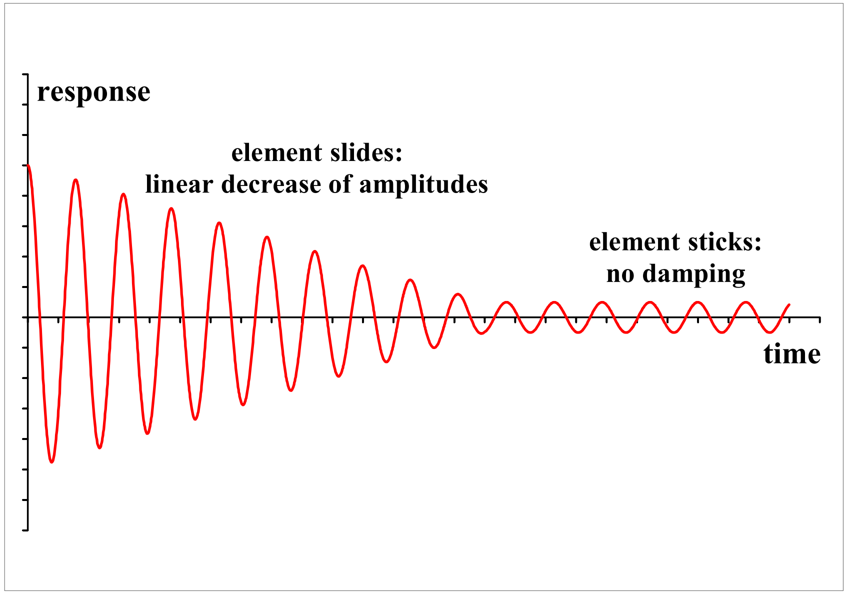
f = f(normal force, friction coefficient)

Properties of Friction Damping

nonlinear problem

- sticking and sliding
- dependent on the direction of motion
- dependent on the normal force

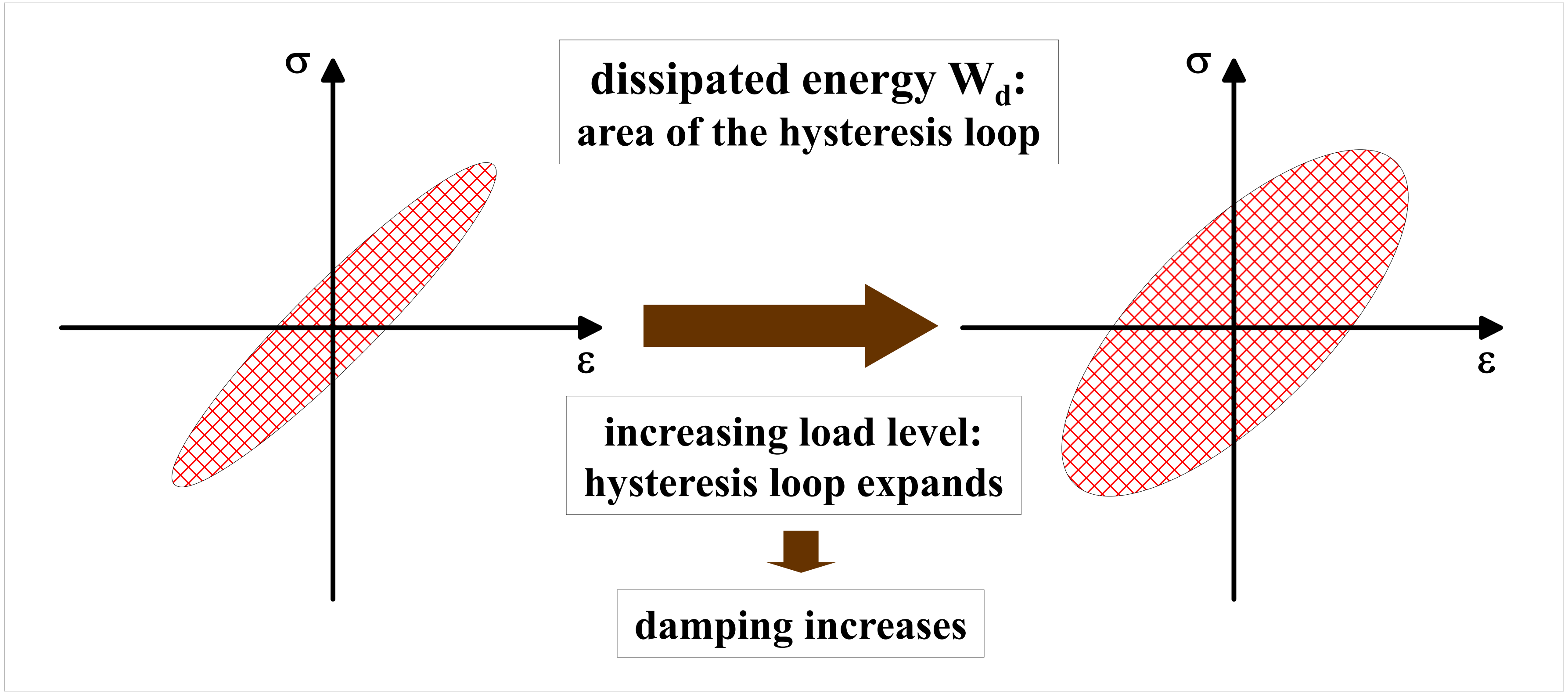
linear decrease of amplitudes



eigenfrequency does not change

Model 2: Hysteretic Damping

Model for material damping: dissipated energy during a loading/unloading cycle is a measure for the damping.



Properties of Hysteretic Damping 1

Damping depends on the strain or *displacement amplitude*:
The higher the load level, the higher the damping.

⇓

nonlinear model

Experiments:
Quadratic dependency of the dissipation work on the amplitude

⇓

$$F_d = K\,|v|\,\mathrm{sign}(\dot{v})$$

⇒ **K = ?**

Properties of Hysteretic Damping 2

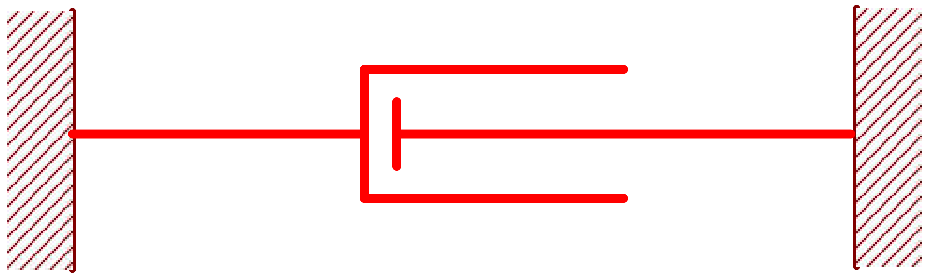
Hysteresis:
Valid only for *stationary harmonic* oscillation processes

What do we do with other, *non-harmonic loads*, e.g. impulse loads, where there are no hysteresis loops?

- **Interesting model with untapped potential.**
- **Still in the research phase.**
- **Not yet with large practical impact.**

Model 3: Viscous Damping

symbolic viscous damping element



General assumption:
The damping force is a function of the *velocity*:

$$\boxed{F_d = F_d(\dot{v})}$$

Specialisation: Weakly Damped Systems

damping is low → **linear viscous damping** ← **NEWTON**

The damping force is proportional to the velocity:

$$F_d = c\dot{v} = F_c$$

c: damping coefficient, simply called "damping".

Properties of Viscous Damping: Energy Dissipation During a Harmonic Cycle

Ratio of dissipated energy: loss factor

$$d = \frac{W_d}{2\pi\, W_e}$$

W_d: dissipated energy
W_e: elastic energy

Question:
What does the loss factor depend on for a *harmonic deformation process*? A harmonic deformation process would produce perfect hysteresis loops. The energy dissipation due to material damping associated with these loops should increase with growing deformation. The question is: does viscous damping show this characteristic, or not? Let's see .....

Harmonic deformation process:

$$v = v_0 \sin \Omega t$$ $\Rightarrow$ $$\dot{v} = v_0 \Omega \cos \Omega t$$

Dissipated energy:

$$W_d = \oint F_d \, dv = \oint F_d \frac{dv}{dt} dt = \int_0^T (c\,\dot{v})\,\dot{v}\,dt = c \int_0^{2\pi/\Omega} v_0^2\,\Omega^2 (\cos \Omega t)^2 dt$$

$\Rightarrow$ $$W_d = \pi c \Omega v_0^2$$

Elastic energy:

$$W_e = \int_0^{v_0} F_k \, dv = \int_0^{v_0} k \, v \, dv = \frac{1}{2} k \, v_0^2$$

$$d = \frac{\pi \, c \, \Omega \, v_0^2}{2 \pi \frac{1}{2} k \, v_0^2}$$

$$d = \frac{c \, \Omega}{k}$$

The relative energy dissipation is frequency-dependent, but not amplitude-independent!

Damping model is in violation of experimental data!

Summary: Viscous Damping

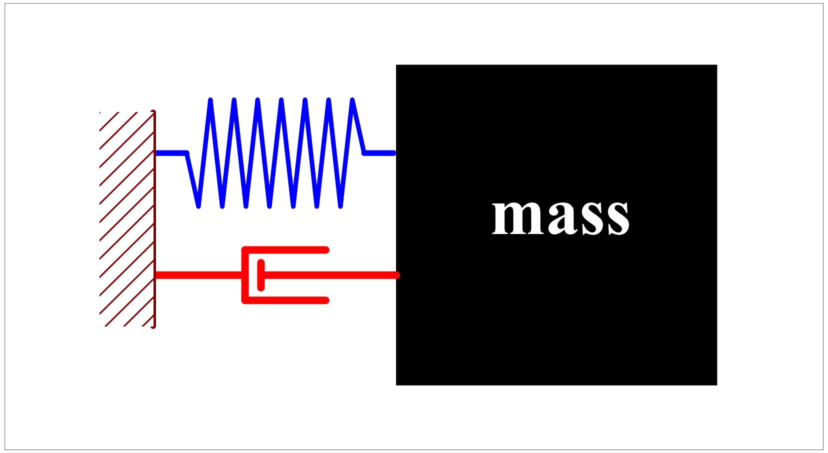
Despite theoretical deficiencies:

Viscous damping is generally accepted as the only practically feasible damping model:

- **Mathematically simple,**
- **Damping can be easily determined by experiments,**
- **Relatively good description of the GLOBAL behaviour.**

We will always assume linear viscous damping!

Total Spring/Damping Law for Viscous Damping



The total force exerted by the mass onto a supporting structure is made up of the *elastic force* plus *the damping force*:

$$F_{tot} = F_k + F_c$$

These two forces do not develop their maximum values simultaneously at the same time instances. The maximum value of the total force therefore is smaller than the sum of the two individual maximum values.

The 2 forces have a *phase angle* of $\pi/2$:

$$F_k = k\, v_0 \sin \Omega t$$

$$F_c = c\, \Omega\, v_0 \cos \Omega t$$

F_k **maximum** → F_c **is zero**

F_c **maximum** → F_k **is zero**

Total force:

$$F_{tot} = k\, v_0 \sin \Omega t + c\, \Omega\, v_0 \cos \Omega t$$

Maximum value of the transmitted force:

$$F_{max} = v_0 \sqrt{k^2 + c^2 \Omega^2}$$

Next Step

- **Free vibration of damped systems:**
  - **Eigenfrequencies?**
  - **Response in the time domain?**

- **Values for the damping:**
  - **Experimental determination of the damping?**
  - **Data range for the damping?**